Basic Electrical Theory

Impedance

PJM State & Member Training Dept.

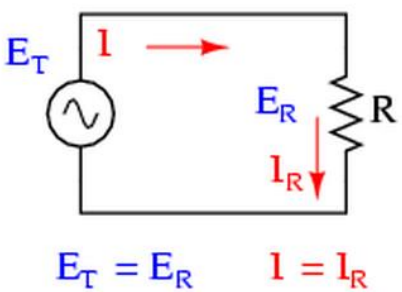
Objectives

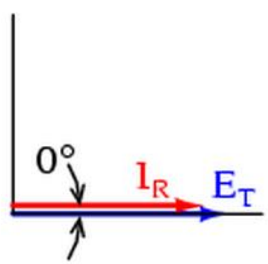
- Identify the components of Impedance in AC Circuits
- Calculate the total Impedance in AC Circuits
- Identify the characteristics of Phase Angles

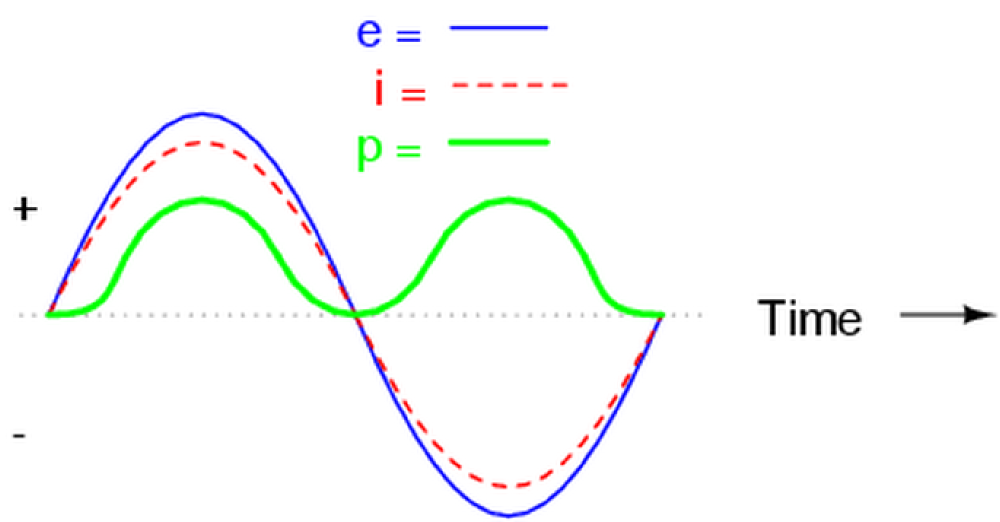
Components of Impedance
AC Circuits

Resistance

- Resistance: $R = \frac{E}{I}$
- A Change in frequency has no effect on resistance
- Current through a resistor and the voltage drop across the resistor are always in phase







Resistance characteristics

- In a purely resistive AC circuit:
  - Voltage and current cycles begin and end at the same time
  - Voltage and current peak values occur at the same time
- Relationship between current and voltage for resistance in an AC circuit is the same as it is in a DC circuit
- Measured values of current and voltage are the RMS or Root Mean Square values of these quantities
- Only resistance consumes power in a circuit

$$P = I_{RMS}\, E_{RMS} \cos \emptyset$$

Question 1

A circuit has a 133 ohm resistor connected to a 24 volt source. Determine the current flowing in the circuit

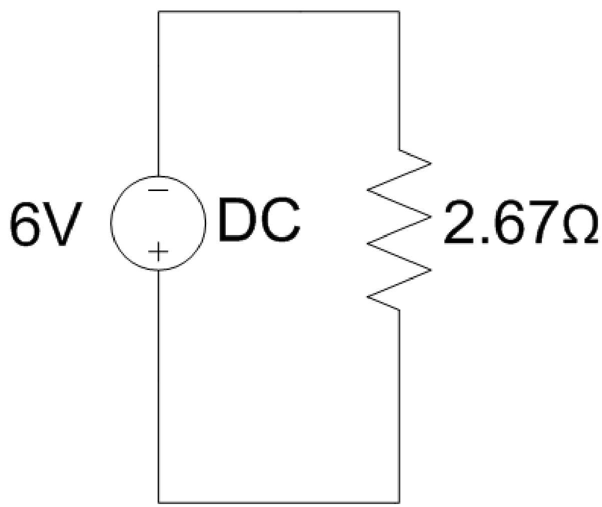
$$E = IR$$

$$\text{b)}\ I = \frac{E_S}{R} = \frac{24\,\text{V}}{133\,\Omega} = 0.18\,\text{A}$$

Question 2

A stereo receiver has a set of speakers, main and remote, for each channel. The speakers for each channel are connected in parallel to the receiver. The AC voltage across the speakers is 6.0 volts and the equivalent resistance is 2.67Ω. Determine the total current supplied by the receiver and the power dissipated in the set of speakers

Question 2



$$I_T = \frac{E_S}{R_T} = \frac{6\,\text{V}}{2.67\,\Omega} = 2.25\,\text{A}$$

$$P_T = (6\,\text{V})(2.25\,\text{A}) = 13.5\,\text{W}$$

Inductive Reactance characteristics

- An inductor’s basis of operation is Faraday’s law of electromagnetic induction
- An inductor develops a voltage that opposes a change in current
- Does not convert electrical energy into heat energy

Inductive Reactance characteristics

- It is the result of induced voltage in a coil by the moving magnetic field created by current flow
- Current must be changing for voltage to be induced
- An inductor allows just enough current flow to produce a voltage equal to but opposing the source voltage
- Inductive reactance (XL) is measured in ohms and determines how much rms current exists in an inductor for a given rms voltage across the inductor

Inductive Reactance

- Average power and average energy used by a inductor in an AC circuit is zero
  a) When the voltage and current product is positive, the inductor is returning energy
  b) When the voltage and current product is negative, energy is delivered to the inductor

Inductive Reactance

- Ohm's Law and inductive reactance:

$$\mathrm{E} = I \times X_L$$

where, $$X_L = 2\pi f L$$

E and I = RMS values for voltage and current

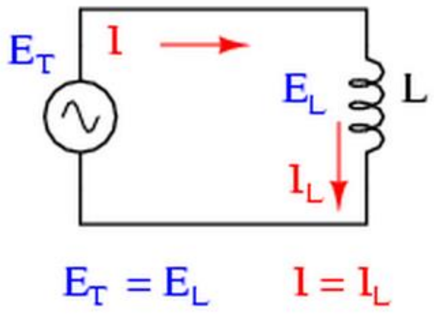
f = frequency (hertz)

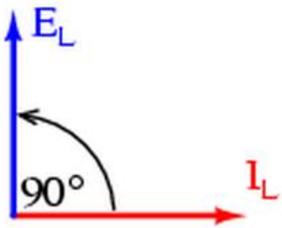
L = inductance (henry)

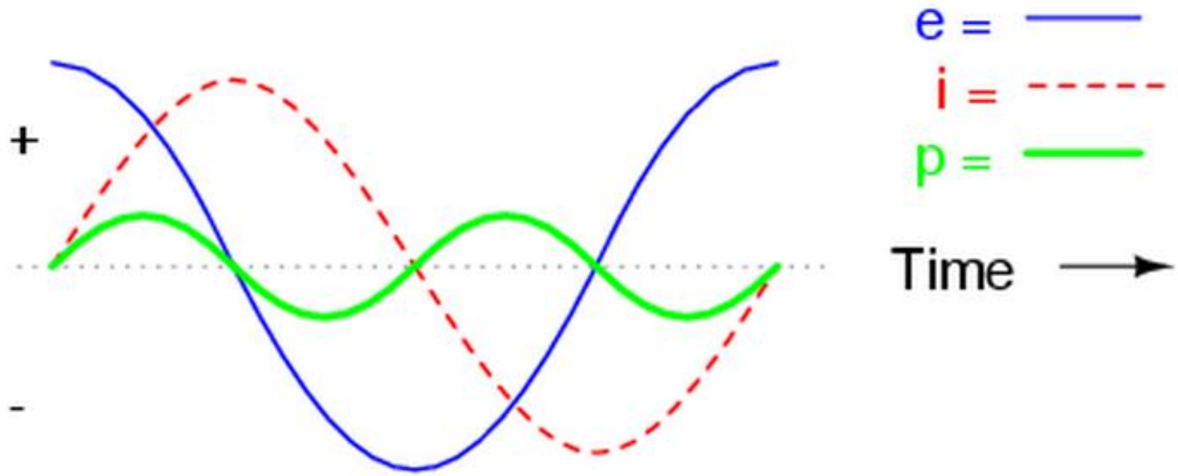
- Increasing frequency increases inductive reactance.
- As frequency increases, current changes more rapidly increasing the value of induced voltage.

Inductive Reactance

- In a purely inductive circuit, voltage leads the current by 90 degrees







Question 3

Calculate the inductive reactance of a circuit having a pure inductance of 50 millihenries with a frequency of 60 Hz.

$$X_L = 2\pi fL = 2\pi(60\,\text{Hz})(0.05\text{H}) = 18.85\,\Omega$$

Question 4

Calculate the inductive reactance of a 2 henry inductance at 60 Hz.

$$X_L = 2\pi fL$$

$$X_L = 2\pi(60)(2)$$

$$X_L = 754\Omega$$

Question 5

A 0.047 H inductor is wired across the terminals of a generator that has a voltage of 2.1 volts and supplies a current of 0.023 amps. What is the frequency of the generator?

$$X_L = \frac{2.1\,\text{V}}{0.023\,\text{A}} = 91.3\,\Omega$$

$$X_L = 2\pi f \text{L}$$

$$91.3\,\Omega = 2\pi f\,(0.047\,\text{H})$$

$$\text{f} = \frac{91.3\,\Omega}{2\pi(0.047\,\text{H})} = \frac{91.3\,\Omega}{0.295} = 309.2\,\text{Hz}$$

Question 6

An 8.2 mH inductor is connected to an AC generator that is operating at 10.0 V and 620 Hz. What is the current supplied by the generator?

$$X_L = 2\pi f L = 2\pi(620\,\text{Hz})(0.0082\,\text{H}) = 31.94\,\Omega$$

$$I_P = \frac{E}{X_L} = \frac{10\,\text{V}}{31.94\,\Omega} = 0.313\,\text{A}$$

Capacitive Reactance

- Ohm's Law and capacitive reactance:

$$I = \frac{E}{X_C}$$

$$X_C = \frac{1}{2\pi f C}$$

where,

E and I = RMS values for voltage and current

f = frequency (hertz)

C = capacitance (farads)

- Increasing frequency decreases capacitive reactance
- As frequency decreases, capacitive reactance becomes infinitely large, and a capacitor provides so much opposition to the motion of charges that there is no flow of current

Capacitive Reactance characteristics

- In an AC circuit containing a capacitor, the polarity of the voltage continually reverses switching back and forth with the electrical charges also surging back and forth
- This constitutes an alternating current with charge flowing continuously
- A capacitor controls the current in an AC circuit by storing energy that produces voltage in a capacitor
- Capacitive reactance (X_C) is measured in ohms and determines how much rms current exists in a capacitor in response to a given rms voltage across the capacitor

Capacitive Reactance characteristics

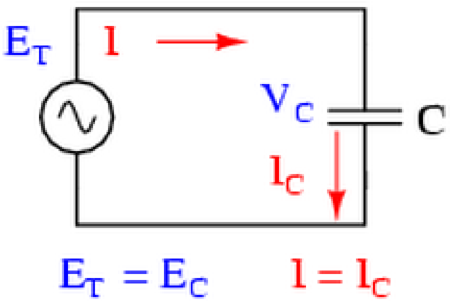
- Does not convert electrical energy into heat energy
- It is the result of the capacitor storing energy that produces a voltage that opposes the source voltage and controls current
- Average power and average energy used by a capacitor in an AC circuit is zero

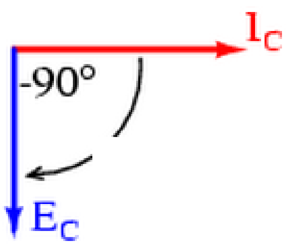
    a) When the voltage and current product is positive, energy is delivered to the capacitor

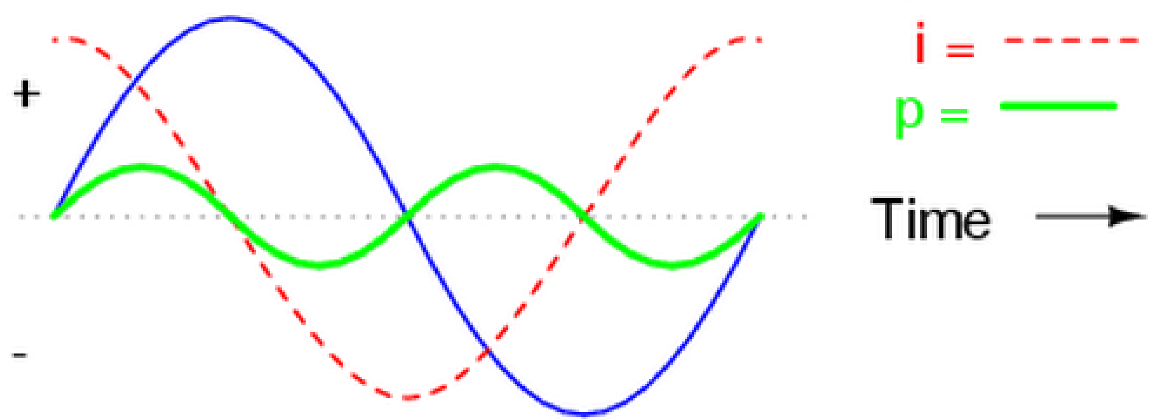
    b) When the voltage and current product is negative, the capacitor is returning energy

Capacitive Reactance

- In a purely capacitive circuit, current leads the voltage by 90 degrees







Capacitive Reactance

- Capacitors are used by utilities for:
  - voltage regulation
  - power factor correction
  - inductance reduction
  - measuring devices for protection systems
  - communications for power line carriers
  - filters for undesirable high frequency signals

Question 7

Calculate the capacitive reactance of a circuit having a capacitor of 400 microfarads with a frequency of 60 Hz.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60\,\text{Hz})(0.0004\,\text{F})} = 6.63\,\Omega$$

Question 8

In a circuit, the capacitance of a capacitor is 1.65 microfarads, and the rms voltage of the generator is 50 volts. What is the rms current in the circuit when the frequency is (a) 2.00 x 10^2 Hz and (b) 4.50 x 10^3 Hz?

$$\text{a)}\ X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(200\ \text{Hz})(0.00000165\ \text{F})} = 482.3\ \Omega$$

$$I_{RMS} = \frac{E_{RMS}}{X_C} = \frac{50\ \text{V}}{482.3\ \Omega} = 0.104\ \text{A}$$

$$\text{b)}\ X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(4500\ \text{Hz})(0.00000165\ \text{F})} = 21.44\ \Omega$$

$$I_{RMS} = \frac{E_{RMS}}{X_C} = \frac{50\ \text{V}}{21.44\ \Omega} = 2.33\ \text{A}$$

Question 9

Calculate the current through a 1 μfarad capacitor, the effective value of voltage applied across the capacitor is 100 volts, and the frequency is 60 Hz.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60\ \mathrm{Hz})(.000001\ \mathrm{F})} = 2652.6\ \Omega$$

$$I = \frac{E}{X_C} = \frac{100\ \mathrm{V}}{2652.6\ \Omega} = 0.0377\ \mathrm{A}$$

Total Impedance
AC Circuits

Total Impedance

- The impedance (Z) of an AC circuit is a complex sum of resistance (R) and net reactance (XL- XC)
- Impedance usually represented in polar form, with a magnitude and an angle ($Z \angle \theta$)
- Impedance is the total opposition to the flow of charge in an AC circuit
- A right triangle, called the impedance triangle is used to illustrate the relationship between AC resistance, reactance, and impedance

Total Impedance

- Impedance (Z) is measured in ohms and defined as:

$$Z = \sqrt{R^2 + X_T^2}$$

Where:

$X_T = X_L - X_C$

R = Resistance

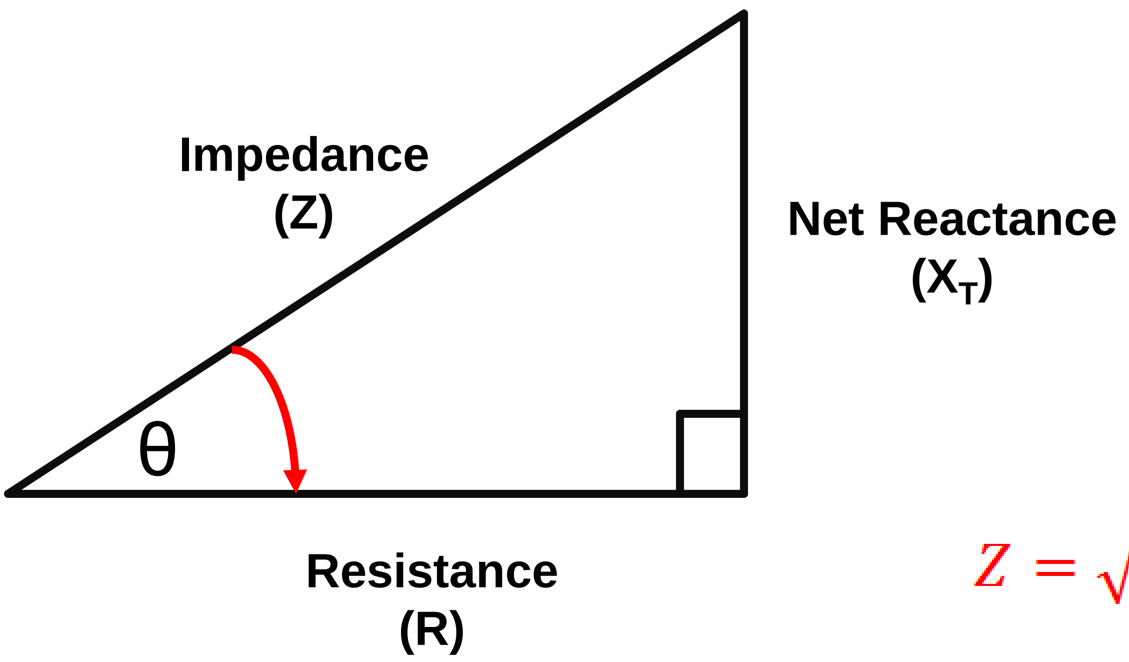
X_T = Total Reactance

X_L = Inductive Reactance

X_C = Capacitive Reactance

- X_L and X_C are 180^o out of phase

Total Impedance



$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \frac{R}{\cos\theta}$$

$$Z = \frac{X_T}{\sin\theta}$$

Question 10

Given a RL circuit with a resistance of 5 ohms and a 37.68 ohm inductance at 60 Hz AC, find the (a) impedance (Z) and, (b) angle theta

$$\text{a) } Z = \sqrt{R^2 + X_T{}^2} = \sqrt{5^2 + 37.68^2} = \sqrt{1444.78} = 38.01\,\Omega$$

$$\cos\theta = \frac{R}{Z}$$

$$\text{b) } \theta = \cos^{-1}\frac{R}{Z} = \cos^{-1}\frac{5}{38.01} = \cos^{-1}(0.132) = 82.44^\circ$$

Question 11

Given a RLC circuit where R is 5 ohms, X_L = 37.68 ohms, and X_C is 2.65 ohms, find the (a) impedance and (b) the angle theta

$$X_T = X_L - X_C = 37.68\,\Omega - 2.65\,\Omega = 35.03\,\Omega$$

$$\text{a) } Z = \sqrt{R^2 + X_T{}^2} = \sqrt{5^2 + 35.03^2} = \sqrt{1252.1} = 35.39\,\Omega$$

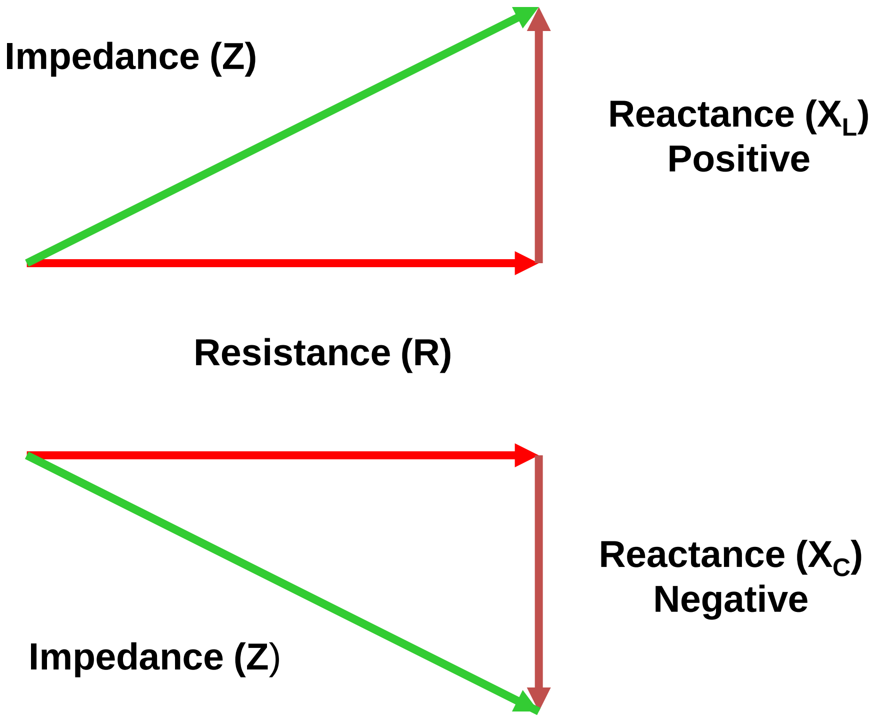
$$\text{b) } \theta = \cos^{-1}\frac{R}{Z} = \cos^{-1}\frac{5\,\Omega}{35.39\,\Omega} = \cos^{-1}(0.1413) = 81.88^\circ$$

Total Impedance

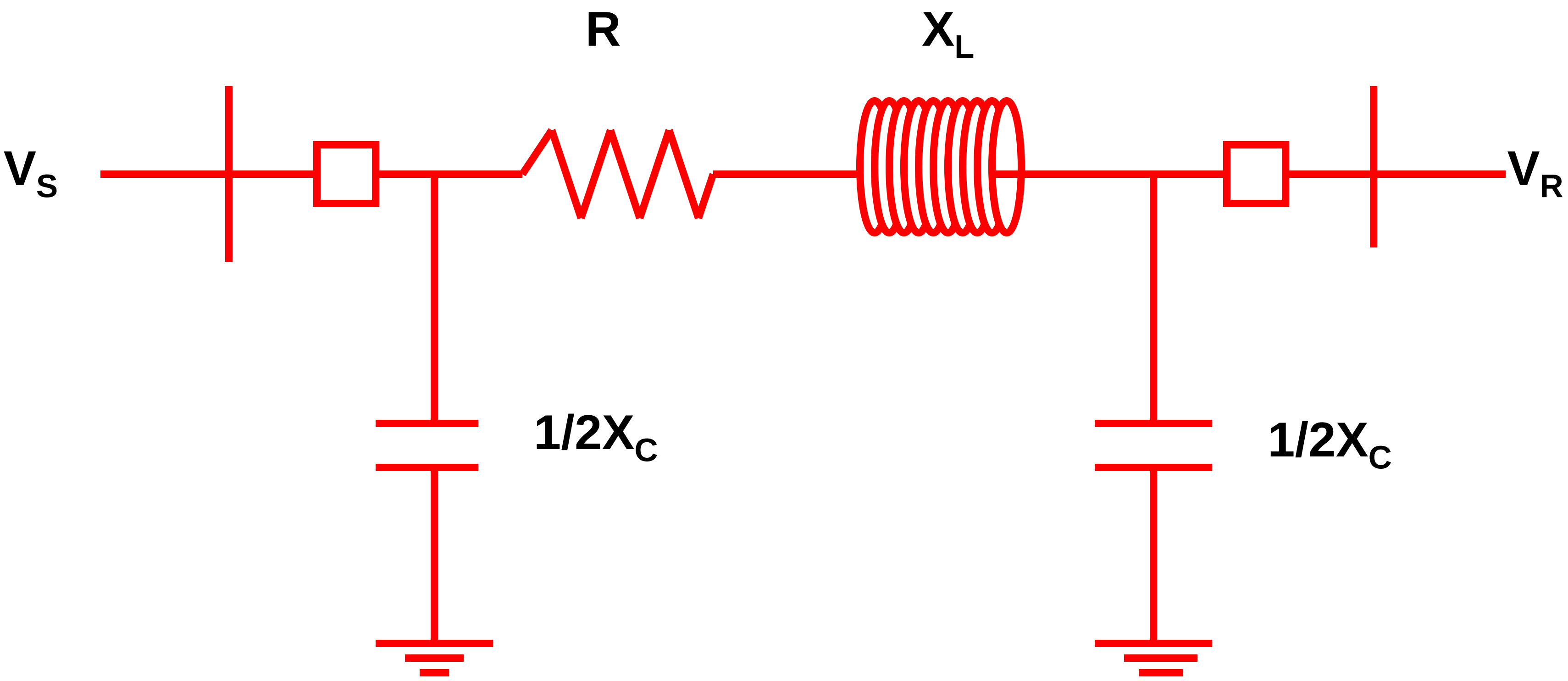
$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \frac{R}{\cos\theta}$$

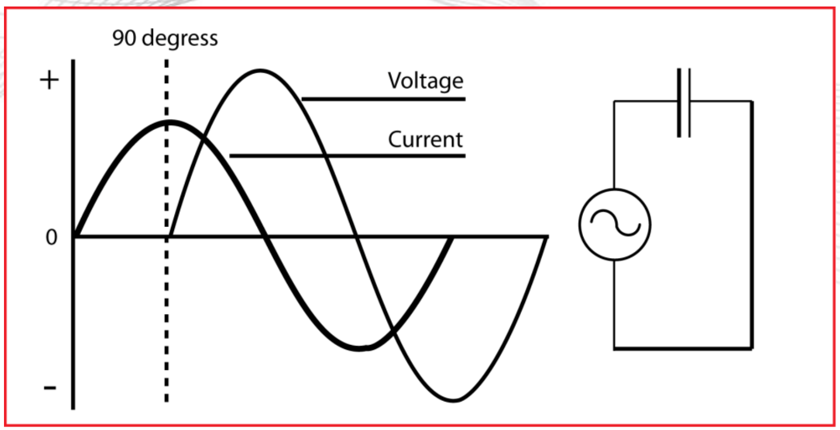
$$Z = \frac{X_T}{\sin\theta}$$



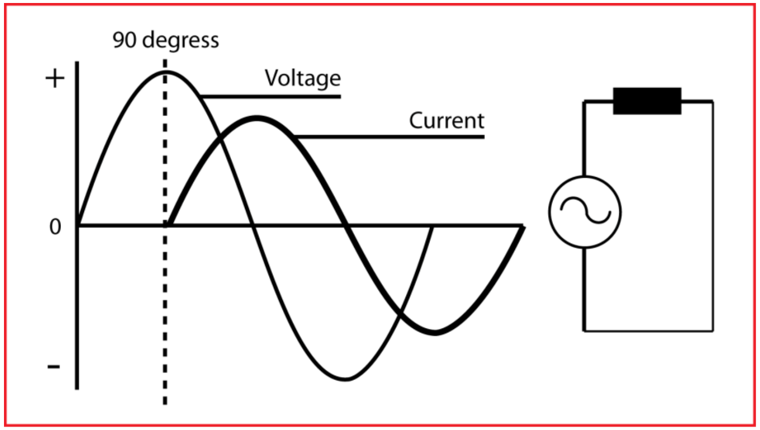
Total Impedance

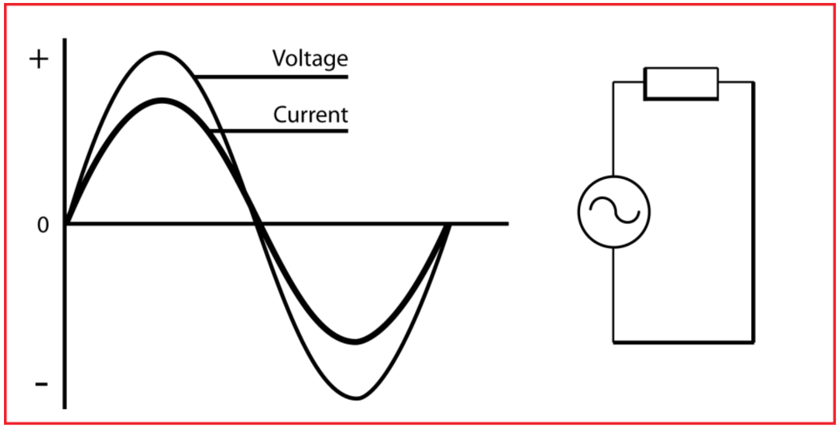


Values will depend on a line's length, cross-sectional area, and conductor spacing



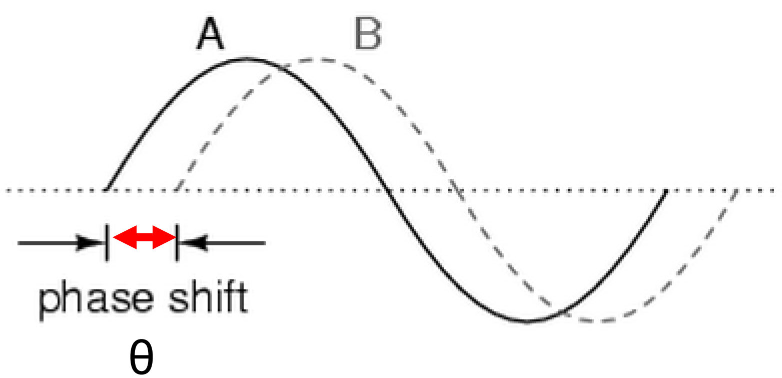
Phase Angle
Part 3

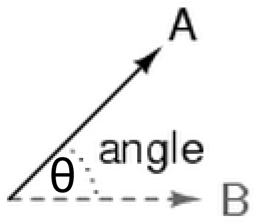




Phase Angle

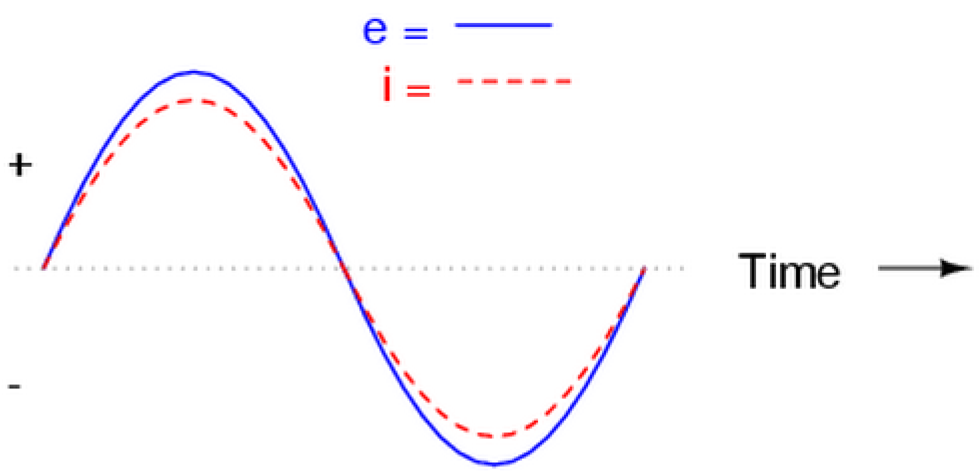
- Phase angle is defined as the angular separation between two phasors
- The spacing between the zero crossings of two waveforms also illustrates the phase angle (θ) of the circuit





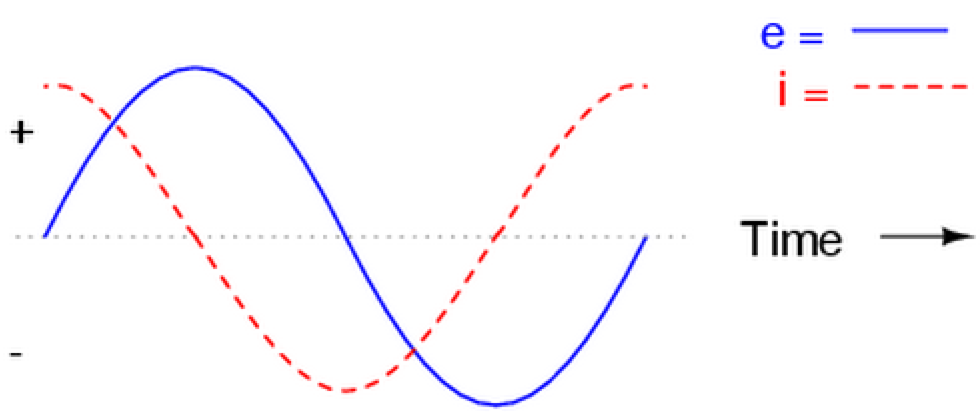
Phase Angle

- The phase angle of a circuit is directly related to the impedance of the circuit
- For a purely resistive circuit, voltage and current will be in phase, and the phase angle will be zero



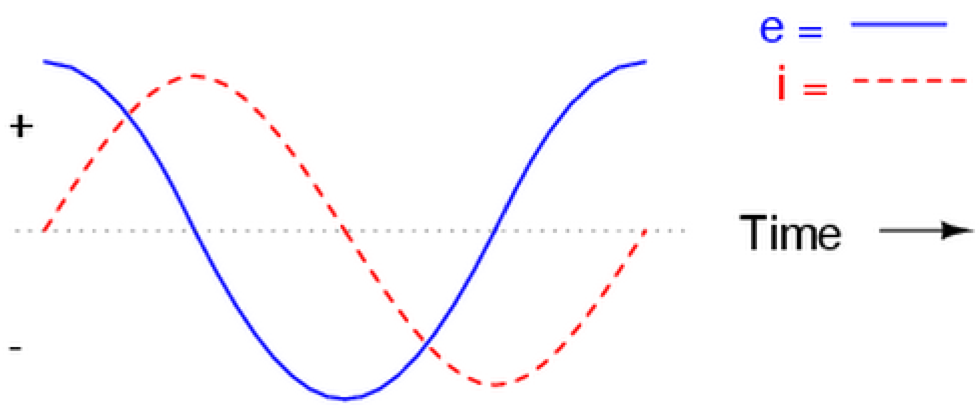
Phase Angle

- A circuit has a leading phase angle when the current wave leads the voltage wave
- This occurs when the circuit is predominantly capacitive because of the energy storage of the electric field



Phase Angle

- A circuit has a lagging phase angle when the current wave lags behind the voltage wave
- This occurs when the circuit is predominantly inductive because of the energy storage of the magnetic field



Summary

- Identified the components of Impedance in AC Circuits
- Calculated the total Impedance in AC Circuits
- Identified the characteristics of Phase Angles

Questions?

LabVolt Exercises

- Do LabVolt exercises 5.2 and 5.4

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For additional detailed information on any of the topics discussed, please refer to the appropriate PJM manual which can be found by accessing: http://www.pjm.com/documents/manuals.aspx