Introduction

This document is intended to provide education on the price formation implications of the nesting or unnesting of PJM's reserve services by walking through a few simple numerical examples. PJM is providing this document to be responsive to stakeholder questions about the implications of reserve nesting/unnesting on energy price formation as well as to illustrate the incentive compatibility between energy and reserve prices that result from the co-optimization. These examples are not intended to be indicative of actual market clearing outcomes but simply to build a fundamental understanding of outcomes of the co-optimization. This document is also not intended to provide a comprehensive overview of how the optimization would be structured or to provide a detailed explanation of the unnested design. An in-depth description of the product nesting proposed in PJM's package is provided in *RCSTF Supplement: Product Nesting and Resource Level Constraints* and a detailed mathematical description of the optimization and incentive compatibility has been provided in *RCSTF Supplement: Incentive Compatibility via Interactions between Reserve and Energy Prices.* These two documents are intended to provide more rigorous technical detail and to be complementary with this set of illustrative examples.

The numerical examples provided in this document use only two generators for simplicity. The implied system is lossless and has no congestion, and so there is a single system marginal energy price. In each of the numerical examples, the generator offers and parameters are held constant, and the reserve requirements are fixed. The only difference in each scenario is an increase in load, which results in a tightening of reserve margins, ending with shortage across all reserve services. For simplicity and to better illustrate the implications of the reserve and energy co-optimization, these examples assume that the two generators do not submit reserve offers, and reserve prices are driven only by lost opportunity cost and the reserve penalty factors. Total reserve quantities and the reserve penalty factors are the same between the nested and unnested cases.

Numerical Examples

Example Set Up

Assume there are two generators on a system serving demand. Both are online and capable of providing reserves. There is no congestion on the system and no losses, and therefore there is a single system marginal energy price. There are three reserve services that are cleared, all of them only in the up direction:

- Synchronized Reserves (SR)
- 10-Min Ramp/Uncertainty Reserves (10-Min RUR)
- 30-Min Ramp/Uncertainty Reserves (30-Min RUR)

In the case where the reserve services are nested, SR can meet the SR Requirement, the 10-Min RUR Requirement and the 30-Min RUR Requirement; 10-Min RUR can meet the 10-Min RUR Requirement and the 30-Min RUR requirement; and 30-Min RUR can only meet the 30-Min RUR Requirement. In the case where the reserve services are unnested, SR can only meet the SR Requirement, 10-Min RUR can only meet the 10-Min RUR Requirement and 30-Min RUR can only meet the 30-Min RUR Requirement. The equations for how each reserve service requirement would be met in both the nested and unnested designs are given below.

Nested Reserve Constraints

$$SR >= SR\ Requirement$$

$$SR + 10Min\ RUR >= 10Min\ RUR\ Requirement^{*}$$

$$SR + 10Min\ RUR + 30Min\ RUR >= 30Min\ RUR\ Requirement^{\dagger}$$

Unnested Reserve Constraints

$$SR >= SR\ Requirement$$

$$10Min\ RUR >= 10Min\ RUR\ Requirement$$

$$30Min\ RUR >= 30Min\ RUR\ Requirement$$

The parameters of the two generators in these examples are given in Table 1. For simplicity, the examples assume that these parameters and offers do not change across the load scenarios and that there are no reserve offers (i.e., all reserve costs are driven only by lost opportunity cost).

Table 1: Generator parameters and energy offer

Generator	Economic Minimum	Economic Maximum	Ramp Rate	Energy Offer
G1	20 MW	70 MW	2 MW/min	$5
G2	20 MW	70 MW	1 MW/min	$10

The reserve procurement targets and penalty factors for each of the three reserve services are given in Table 2 for both the nested and unnested case.

Table 2: Reserve requirements and penalty factors

Service	Nested Requirement	Unnested Requirement	Penalty Factor
SR	9 MW	9 MW	$40
10-Min RUR	18 MW	9 MW	$20
30-Min RUR	37 MW	19 MW	$20

Scenario 1: Sufficient Unloaded Headroom

If system load is 80 MW, there is sufficient free, unloaded headroom to meet all the reserve requirements, and all the reserve services clear at $0/MWh.

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* Inclusive of SR Requirement

† Inclusive of 10-Min Requirement

Table 3: Generator assignments

Generator	Energy	SR	10-Min RUR	30-Min RUR
G1	60 MW	9 MW	0 MW	0 MW
G2	20 MW	0 MW	9 MW	19 MW

Table 4: Market clearing prices for both the nested and unnested reserve design

	Market Clearing Prices	
	Nested Reserves	Unnested Reserves
Energy	$5/MWh	$5/MWh
SR	$0/MWh	$0/MWh
10-Min RUR	$0/MWh	$0/MWh
30-Min RUR	$0/MWh	$0/MWh

Scenario 2: Generator 1 Incurs Lost Opportunity Cost

If system load is 90 MW, there is no longer sufficient unloaded headroom to meet all the reserve requirements, and G1 incurs lost opportunity cost. G1's economic operating point would be at its EcoMax of 70 MW, and when it is backed down to provide 8 MW of reserves, it incurs an opportunity cost equal to the difference between its energy offer at $5/MWh and the system marginal energy price of $10/MWh. Hence, the reserve market clearing price reflects this $5/MWh lost opportunity cost. Because the same reserve capability is eligible to provide both SR and 10-Min RUR and there are no reserve offers in these examples, there is price parity between these two reserve market clearing prices as long as they are not both clearing at the penalty factor,‡ and both SR and 10-Min RUR clear at $5/MWh.

Table 5: Generator assignments

Generator	Energy	SR	10-Min RUR	30-Min RUR
G1	62 MW	0 MW	8 MW	0 MW

‡ The explanation and mathematical proof for why this is true is included in *RCSTF Supplement: Incentive Compatibility via Interactions between Reserve and Energy Prices*

G2	28 MW	9 MW	1 MW	19 MW

Table 6: Market clearing prices for both the nested and unnested reserve design

	Market Clearing Prices	
	Nested Reserves	Unnested Reserves
Energy	$10/MWh	$10/MWh
SR	$5/MWh	$5/MWh
10-Min RUR	$5/MWh	$5/MWh
30-Min RUR	$0/MWh	$0/MWh

Scenario 3: One RUR Service Clears at the Reserve Penalty Factor

If the system load increases to 110 MWh, there is no longer sufficient capacity on the system to serve the load and meet all three reserve requirements. The system will therefore clear short of one of the lower value reserve services. In the nested reserve design, the shortage occurs in the 30-Min RUR reserve service because of the hierarchy in the services. In the unnested case, because the 10-Min RUR and 30-Min RUR reserve services have the same penalty factor of $20/MWh, it is more economic to clear more reserves on G2, allowing G1, which has a lower cost, to produce more energy. This is possible because G2 has additional headroom that can be available in 30 minutes beyond the 10-minute reserve capability it can provide. In this case, both the pricing and dispatch outcomes are different between the nested and unnested cases.

In the nested design, the 30-Min RUR clearing price is set based on the $20/MWh reserve penalty factor. 10-Min RUR is not clearing in shortage, and the next incremental MW of 10-Min RUR would come from backing down G1 at a cost of $5/MWh. Because of the nesting, the 10-Min RUR clearing price is the sum of this cost plus the cost of 30-Min RUR, leading to a total market clearing price of $25/MWh. The SR service is not binding, and so the SR market clearing price is also equal to $25/MWh. The system marginal energy price is based on the cost of clearing 1 MW additional on G2. This would come at the cost of 1 MW of additional 30-Min RUR shortage at $20/MWh plus the cost of energy, which is $10/MWh based on G2's energy offer. This leads to a total system marginal energy price of $30/MWh.

In the unnested design, the 10-Min RUR market clearing price is set based on the $20/MWh reserve penalty factor. 30-Min RUR is not clearing in shortage, and the next incremental MW of 30-Min RUR would come from backing down G2 by another MW. This would require ramping up G1 to meet the energy balance, causing an additional MW of 10-Min RUR shortage. The cost of this 10-Min RUR shortage comes at a cost of $20/MWh but because G1 has a lower cost for energy than G2 by $5/MWh, the total cost is $15/MWh, which becomes the 30-Min RUR market clearing price. Synchronized Reserves are also not clearing in shortage, and the next MW of SR would come from incurring 1 MW of additional 10-Min RUR shortage, at a cost of $20/MWh, which leads to a SR market clearing price of $20/MWh. The system marginal energy price is once again based on the cost of clearing 1 MW of energy on the marginal energy resource, but this time on G1. This would come at the cost of 1 MW of additional 10-Min RUR shortage, at the cost of $20/MWh, plus G1's energy offer of $5/MWh. This leads to an energy price of $25/MWh.

Table 7: Generator assignments in the nested design

Generator	Energy	SR	10-Min RUR	30-Min RUR
G1	62 MW	0 MW	8 MW	0 MW
G2	48 MW	9 MW	1 MW	12 MW

Table 8: Generator assignments in the unnested design

Generator	Energy	SR	10-Min RUR	30-Min RUR
G1	69 MW	0 MW	1 MW	0 MW
G2	41 MW	9 MW	1 MW	19 MW

Table 9: Market clearing prices for both the nested and unnested reserve design

	Market Clearing Prices	
	Nested Reserves	Unnested Reserves
Energy	$30/MWh	$25/MWh
SR	$25/MWh	$20/MWh
10-Min RUR	$25/MWh	$20/MWh
30-Min RUR	$20/MWh	$15/MWh

Scenario 4: Both 10-Min RUR and 30-Min RUR Clear at the Reserve Penalty Factor

If the system load increases to 130 MWh, there is no longer sufficient capacity on the system to serve the load and meet all three reserve requirements. The system does not clear any 30-Min RUR and clears short of the full 10-Min RUR Requirement, meaning that both reserve services clear at the $20/MWh penalty factor. Even though the SR and 10-Min RUR reserve services depend on the same physical resource capability, the optimization will always prioritize clearing the SR Requirement and allow the 10-Min RUR Requirement to clear short of its full quantity. In the nested design, this is enforced by the reserve hierarchy. In the unnested design, this happens because the SR penalty factor is higher than the 10-Min RUR penalty factor. While the reserve quantities remain the same, in this scenario, the pricing outcomes are different between the nested and unnested reserve design because two of the reserve services are now clearing at the penalty factor.

In both reserve designs, the 30-Min RUR clearing price is set by the 30-Min RUR penalty factor, $20/MWh. In the nested design, the 10-Min RUR clearing price is set by the sum of the shadow price of the 30-Min RUR constraint ($20/MWh) and the 10-Min RUR constraint ($20/MWh) for a total of $40/MWh. Because the SR service constraint is fully met, it is not binding in the nested design, and its clearing price is the same as the 10-Min RUR clearing price at $40/MWh. In the unnested design, only the 10-Min RUR penalty factor sets the clearing price the 10-Min RUR service and because of the price parity between the 10-Min RUR and SR services, the SR service also clears at $20/MWh.

This difference in reserve clearing prices drives differences in the system marginal energy price. The marginal energy resource in this scenario is G2. If system load was increased by 1 MW, that would lead to 1 MW of additional 10-Min RUR shortage plus the cost of that incremental MWh of energy, at G2's energy offer of $10/MWh. In the nested case, the cost of one additional MW of 10-Min RUR shortage is $40/MWh, leading to a marginal energy price of $50/MWh. In the unnested case, the cost of one additional MW of 10-Min RUR shortage is $20/MWh, leading to a marginal energy price of $30/MWh.

Table 10: Generator assignments

Generator	Energy	SR	10-Min RUR	30-Min RUR
G1	70 MW	0 MW	0 MW	0 MW
G2	60 MW	9 MW	1 MW	0 MW

Table 11: Market clearing prices for both the nested and unnested reserve design

	Market Clearing Prices	
	Nested Reserves	Unnested Reserves
Energy	$50/MWh	$30/MWh
SR	$40/MWh	$20/MWh
10-Min RUR	$40/MWh	$20/MWh
30-Min RUR	$20/MWh	$20/MWh

Scenario 5: All Reserves Clear at the Reserve Penalty Factor

If the system load increases to 135 MWh, there is no longer sufficient capacity on the system to serve the load and meet all three reserve requirements. The system does not clear any 10-Min RUR or 30-Min RUR and clears short of the full SR Requirement, meaning that all three reserve services clear at the penalty factor, which is $20/MWh for 10-Min RUR and 30-Min RUR and $40/MWh for SR. While the reserve quantities remain the same, in this scenario, the pricing outcomes are once again different between the nested and unnested reserve design.

For the nested design, the SR clearing price is the sum of all three reserve penalty factors. For the unnested design, the SR clearing price is only the SR reserve penalty factor.

Once again, this difference in reserve clearing prices drives differences in the system marginal energy price. The marginal energy resource in this scenario is still G2. If system load was increased by 1 MW, that would lead to 1 MW of additional SR shortage plus the cost of that incremental MWh of energy, at G2's energy offer of $10/MWh. In the nested case, the cost of one additional MW of SR shortage is $80/MWh, leading to a marginal energy price of $90/MWh. In the unnested case, the cost of one additional MW of SR shortage is $40/MWh, leading to a marginal energy price of $50/MWh.

Table 12: Generator assignments

Generator	Energy	SR	10-Min RUR	30-Min RUR
G1	70 MW	0 MW	0 MW	0 MW
G2	65 MW	5 MW	0 MW	0 MW

Table 13: Market clearing prices for both the nested and unnested reserve design

	Market Clearing Prices	
	Nested Reserves	Unnested Reserves
Energy	$90/MWh	$50/MWh
SR	$80/MWh	$40/MWh
10-Min RUR	$40/MWh	$20/MWh
30-Min RUR	$20/MWh	$20/MWh

Observations

- Resources are indifferent to providing reserves and energy in both designs because of the co-optimization between energy and reserves. The lost opportunity cost incurred by resources providing reserves rather than energy appears in reserve market clearing prices.
- If the same resources are eligible to provide multiple reserve services, these resources could incur lost opportunity cost for providing one service rather than another. If applicable, this lost opportunity cost will appear in reserve market clearing prices through the co-optimization.

- At the point at which all reserve services are clearing at their respective penalty factors, the hierarchy in the reserve penalty factors will dictate how reserves are cleared. If there are insufficient reserves to meet any of the requirements, all the available reserves will clear for the reserve service with the highest penalty factor.

A Few Practical Notes on the Design

Reserve Offers

These examples assume that the same set of resources are eligible to provide Synchronized Reserves and 10-Min Ramp/Uncertainty Reserves and that in both cases, the resources would be indifferent to providing each service. In practice, this is unlikely to hold true because of the increased performance obligations and consequences for non-performance imposed on resources with a Synchronized Reserve assignment during a Synchronized Reserve Event. PJM anticipates that this will lead resources to submit higher offers for providing Synchronized Reserves than for 10-Min Ramp/Uncertainty Reserves, which is supported by the proposed offer rules. This would mean that there is likely to be price separation between Synchronized Reserves and 10-Min Ramp/Uncertainty Reserves, which was not included in these examples.

Reserve Penalty Factors

For the purpose of explaining the differences in market clearing outcomes based on the co-optimization, these examples use the same penalty factors in both the nested and unnested designs. This allows for an apples-to-apples comparison that illustrates the implications of the nesting/unnesting on price formation. However, in practice, PJM is proposing increasing the Synchronized Reserve penalty price from $850/MWh at the Synchronized Reserve Requirement to $2,100/MWh to better reflect the value of the service and to recognize the fact that Synchronized Reserves will no longer be nested with either Primary and 30-Minute Reserves.