

# RCSTF Supplement: Locational Constraints for Reserve Services

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## 1. Purpose

The goal of this document is to provide details of the locational constraints for the different reserve services that are part of the RCSTF proposal.

**Disclaimer:** Some of the constraints have been simplified for expositional purposes. Moreover, the constraints may be modified in the final implementation based on computational limitations. In case the exposition in this document contradicts the tariff language, the tariff has precedence.

## 2. Preliminaries

This section provides the notation used throughout this appendix.

$\mathcal{T} := \{\text{SR}, 10\text{-RUR-U(p)}, 10\text{-RUR-D(own)}, 30\text{-RUR}, 30\text{-SecR}, 60\text{-Spin}, 60\text{-NonSpin}\}$  denotes the set of reserve products

$\mathcal{S} := \{\text{SR}, 10\text{-RUR-U(p)}, 10\text{-RUR-D(own)}, 30\text{-RUR}, 30\text{-Min-Reserves}, \text{EG}, \text{DASR}\}$  denotes the set of reserve services

$\mathcal{S}^{\text{RUR}} \subset \mathcal{S}$  denotes the subset of RUR services (i.e., 10-Min RUR Up, 10-Min RUR Down, and 30-Min RUR)

$\mathcal{G}$  denotes the set of nodes with resources that can provide baseload or reserve services

$\mathcal{R}$  denotes the set of nodes with renewable generation (wind or solar)

$L$  denotes the set of load nodes

$\mathcal{N}$  denotes the set of all nodes in a network

$\mathcal{M}$  denotes the set of all transmission facilities with modeled constraints

$g_i(t) \in \mathfrak{R}_+$  is the energy dispatch of unit  $i$  at time  $t$

$r_i^{\text{type}}(t) \in \mathfrak{R}_+$  is the reserve assignment of unit  $i$  of a given reserve product

$\text{type} \in \mathcal{T}$  at time  $t$ ; e.g.,  $r_i^{10\text{-RUR-U}}(t)$  is the 10-min RUR (Up) assignment for unit  $i$  at time  $t$

For a transmission line  $m$ ,  $\mathcal{L}_m$  denotes the limit on power flow (symmetric flow limits are assumed for expositional simplicity)

$\text{DF}_{k,m}^{\text{ax}}$  denotes the shift factor from node  $k$  to line  $m$

$\ell_k(t)$  denotes the forecast demand at load node  $k$  for time  $t$

$g_k^S(t), g_k^W(t)$  denote the forecast solar generation and forecast wind generation, respectively, at node  $k$  for time  $t$

The  $\tau$ -min forward ramp at load node  $k$  following target  $t_0$  is defined as

$$\Delta_k^L(t_0, \tau) := \ell_k(\tau) - \ell_k(t_0) \quad (1)$$

The  $\tau$ -min forward ramp due to solar generation at node  $k$  following target  $t_0$  is defined as

$$\Delta_k^S(t_0, \tau) := g_k^S(\tau) - g_k^S(t_0) \quad (2)$$

The  $\tau$ -min forward ramp due to wind generation at node  $k$  following target  $t_0$  is defined similarly and denoted  $\Delta_k^W(t_0, \tau)$

The expected systemwide netload ramp at time  $t$  for the 10-Min RUR Up service is given by

$$\mathbb{E}_t[10\text{-RUR-U}] = \sum_k \Delta_k^L(t, 10) - \Delta_k^S(t, 10) - \Delta_k^W(t, 10), \quad (3)$$

with the possibility of both positive values (when the netload is ramping up) and negative values (when the netload is ramping down). The expected systemwide netload ramp for the 10-Min RUR (Down) service is the negative of the ramp for the up service:

$$\mathbb{E}_t[10\text{-RUR-D}] = -\mathbb{E}_t[10\text{-RUR-U}] \quad (4)$$

The expected systemwide netload ramp at time  $t$  for the 30-Min RUR service is given by

$$\mathbb{E}_t[30\text{-RUR}] = \left( \sum_k \Delta_k^L(t, 30) - \Delta_k^S(t, 30) - \Delta_k^W(t, 30) \right) - \mathbb{E}_t[10\text{-RUR-U}] \quad (5)$$

$rs_k^S(t_0, \tau), rs_k^W(t_0, \tau), rs_k^L(t_0, \tau) \in [0, 1]$  denote the ramp contributions of node  $k$  to the system-wide forecast ramp of solar, wind, and load respectively, till  $\tau$  minutes following time  $t_0$

$$rs_k^S(t_0, \tau) := \frac{\Delta_k^S(t_0, \tau)}{\sum_k \Delta_k^S(t_0, \tau)} \quad (6.1)$$

$$rs_k^W(t_0, \tau) := \frac{\Delta_k^W(t_0, \tau)}{\sum_k \Delta_k^W(t_0, \tau)} \quad (6.2)$$

$$rs_k^L(t_0, \tau) := \frac{\Delta_k^L(t_0, \tau)}{\sum_k \Delta_k^L(t_0, \tau)} \quad (6.3)$$

$Q^{\text{type}} \in \mathfrak{R}_+$  denotes the total systemwide MWs cleared on the demand curve for reserve service type  $\in \mathcal{S}$

$U^{\text{type}}(t) \in \mathfrak{R}$  denotes the total systemwide MWs cleared for uncertainty on the demand curve of the RUR service type  $\in \mathcal{S}^{\text{RUR}}$  at time  $t$ . It is related to  $Q$  as follows: for type  $\in \mathcal{S}^{\text{RUR}}$ ,

$$U^{\text{type}}(t) = Q^{\text{type}}(t) - \max(\mathbb{E}_t[\text{type}], 0) \quad (7)$$

$\text{SUF}_\tau, \text{WUF}_\tau, \text{LUF}_\tau \in [0, 1]$  denote the solar uncertainty factor, wind uncertainty, and load uncertainty factor, which are the contribution factors of solar resources, wind resources, and load, respectively, to the system-level uncertainty requirement for a  $\tau$ -Min RUR service with  $\tau \in \{10, 30\}$

$u_k^{L, \text{type}}(t)$  is the allocation of the systemwide uncertainty MWs  $U^{\text{type}}(t)$  to the load node  $k$  for a reserve service of type  $\in \mathcal{S}^{\text{RUR}}$  based on nodal ramp share and LUF

$u_k^{S, \text{type}}(t), u_k^{W, \text{type}}(t)$  are the allocation of the systemwide uncertainty MWs  $U^{\text{type}}(t)$  for a reserve service of type  $\in \mathcal{S}^{\text{RUR}}$  to the node  $k$  corresponding to the solar and wind generation uncertainties, respectively

For a node  $k$ :  $\text{INC}_k, \text{DEC}_k, \text{UTC}_k^{\text{source}}, \text{UTC}_k^{\text{sink}}$  denote the virtual INC MWs, DEC MWs, UTC MWs for source node transactions, and UTC MWs for sink node transactions, respectively

### 3. Transmission Line Constraints for Energy

Before introducing the locational constraints on reserves, we recall the transmission line constraints on energy.

**Real Time Market (RTM).** For a transmission line  $m$  at time  $t$ , we have,

$$-\mathcal{L}_m \leq \sum_{i \in \mathcal{G}} \text{DF}_{i,m}^{\text{ax}} g_i(t) + \sum_{j \in \mathcal{R}} \text{DF}_{j,m}^{\text{ax}} (g_j^S(t) + g_j^W(t)) - \sum_{k \in \mathcal{L}} \text{DF}_{k,m}^{\text{ax}} \ell_k(t) \leq \mathcal{L}_m \quad (8)$$

We can write this more succinctly as

$$-\mathcal{L}_m \leq \sum_{k \in \mathcal{N}} \text{DF}_{k,m}^{\text{ax}} (g_k(t) + g_k^S(t) + g_k^W(t) - \ell_k(t)) \leq \mathcal{L}_m \quad (9)$$

**Day-ahead Market (DAM).** For a transmission line  $m$  at time  $t$ , we have the following modification to (9) in DAM,

$$\begin{aligned} -\mathcal{L}_m \leq \sum_{k \in \mathcal{N}} \text{DF}_{k,m}^{\text{ax}} (g_k(t) + g_k^S(t) + g_k^W(t) + \text{INC}_k(t) + \text{UTC}_k^{\text{source}}(t) \\ - \ell_k(t) - \text{DEC}_k(t) - \text{UTC}_k^{\text{sink}}(t)) \leq \mathcal{L}_m \end{aligned} \quad (10)$$

Let  $\lambda_m^e(t)$  denote the shadow price of the constraint on energy for transmission line  $m$  at time  $t$ .

## 4. DASR and EG

Reserves that are assigned only in the day-ahead market will only have an RTO-wide procurement with no locational constraints. Therefore, there will be no locational price separation for these reserve products (i.e., the same clearing price applies across the RTO).

## 5. Contingency Reserves

The RCSTF proposal includes two contingency reserve products: 10-Min synchronized reserves (SR) and the offline 30-Min secondary reserves (SecR); the latter serve as backfill for the former.

Both types of contingency products are proposed to be procured based on locational constraints. The locational constraints imposed for the two products may be different. For SR, the constraints are imposed to prevent potential Interconnection Reliability Operating Limit (IROL) violations. For SecR, the constraints may include lines at risk for prolonged violation following a unit loss. For both products, the proposal includes ensuring the deliverability of the products against the loss of the unit that is providing the largest relief from congestion/violation for each modeled constraint.<sup>1</sup>

### 5.1. Synchronized Reserves

In RTM, the flow  $F_m^{\text{SR}}(t)$  on a network constraint  $m$  at time  $t$  including SR is given by,

$$F_m^{\text{SR}}(t) := \sum_{k \in \mathcal{N}} \text{DF}_{k,m}^{\text{ax}} (g_k(t) + r_k^{\text{SR}}(t) + g_k^S(t) + g_k^W(t) - \ell_k(t)) \quad (11)$$

In DAM, the flow will include additional terms for virtual transactions as follows:

$$F_m^{\text{SR}}(t) := \sum_{k \in \mathcal{N}} \text{DF}_{k,m}^{\text{ax}} (g_k(t) + r_k^{\text{SR}}(t) + g_k^S(t) + g_k^W(t) - \ell_k(t)(t))$$

<sup>1</sup>This appendix provides one methodology to identify the loss of the unit providing the largest relief; the exact implementation may differ from this methodology.

$$+INC_k(t) + UTC_k^{\text{source}}(t) - DEC_k(t) - UTC_k^{\text{sink}}(t) \quad (12)$$

To identify the unit with the largest flow in either direction, we introduce the variables  $\xi_m^+$  and  $\xi_m^-$ , with the following constraints:  $\forall k \in \mathcal{N}$ ,

$$\xi_m^+ \geq DF_{k,m}^{\text{ax}} (g_k(t) + r_k^{\text{SR}}(t) + g_k^S(t) + g_k^W(t)), \quad (13.1)$$

$$\xi_m^- \leq DF_{k,m}^{\text{ax}} (g_k(t) + r_k^{\text{SR}}(t) + g_k^S(t) + g_k^W(t)) \quad (13.2)$$

The locational constraint for SR corresponding to the transmission facility  $m$  is then given by,

$$F_m^{\text{SR}}(t) - \xi_m^- \leq \mathcal{L}_m, \quad (14.1)$$

$$F_m^{\text{SR}}(t) - \xi_m^+ \geq -\mathcal{L}_m \quad (14.2)$$

Let  $\lambda_m^{\text{SR}^a}(t)$  and  $\lambda_m^{\text{SR}^b}(t)$  denote the shadow prices corresponding to (14.1) and (14.2), respectively.

## 5.2. 30-Min Secondary Reserves

In RTM, the flow  $F_m^{\text{SecR}}(t)$  on a transmission constraint  $m$  at time  $t$  including SR and the offline 30-Min SecR is given by,

$$F_m^{\text{SecR}}(t) := \sum_{k \in \mathcal{N}} DF_{k,m}^{\text{ax}} (g_k(t) + r_k^{\text{SR}}(t) + r_k^{\text{30-SecR}}(t) + g_k^S(t) + g_k^W(t) - \ell_k(t)) \quad (15)$$

Note that, since 30-Min SecR products are offline, we have the following orthogonal relationship:

$$r_i^{\text{30-SecR}}(t) \perp g_i(t) + r_i^{\text{SR}}(t), \quad (16)$$

that is, at any given time, only one side of (16) can be non-zero for any standalone unit  $i \in \mathcal{G}$ .

In DAM, the flow in (15) is modified similar to (12). Equation (13) admits the following modification to include 30-Min SecR, with the variables  $\zeta_m^+$ ,  $\zeta_m^-$  replacing  $\xi_m^+$ ,  $\xi_m^-$ :

$$\zeta_m^+ \geq DF_{k,m}^{\text{ax}} (g_k(t) + r_k^{\text{SR}}(t) + r_k^{\text{30-SecR}}(t) + g_k^S(t) + g_k^W(t)), \quad (17.1)$$

$$\zeta_m^- \leq DF_{k,m}^{\text{ax}} (g_k(t) + r_k^{\text{SR}}(t) + r_k^{\text{30-SecR}}(t) + g_k^S(t) + g_k^W(t)) \quad (17.2)$$

The locational constraint for 30-Min SecR corresponding to the transmission facility  $m$  is then given by,

$$F_m^{\text{SecR}}(t) - \zeta_m^- \leq \mathcal{L}_m, \quad (18.1)$$

$$F_m^{\text{SecR}}(t) - \zeta_m^+ \geq -\mathcal{L}_m \quad (18.2)$$

Let  $\lambda_m^{\text{SecR}^a}(t)$  and  $\lambda_m^{\text{SecR}^b}(t)$  denote the shadow prices corresponding to (18.1) and (18.2), respectively.

## 6. Ramping/Uncertainty Reserves (RUR)

These SCED-dispatchable online reserve products are proposed to be procured locationally based on transmission line constraints. We begin by discussing the locational constraints in RTM.

**10-Min RUR (Up).** Before we can introduce the constraint, we need to first allocate the uncertainty  $U^{10\text{-RUR-U}}$  cleared on the demand curve to the load and renewable nodes as follows. For a load node  $k$ , the uncertainty allocation in MW is given by:

$$u_k^{L,10\text{-RUR-U}}(t) := \text{rs}_k^L(t, 10) \times \text{LUF}_{10} \times U^{10\text{-RUR-U}}(t) \quad (19)$$

Similarly, for the solar and wind nodes  $k$ , the nodal uncertainty allocations are, respectively,

$$u_k^{S,10\text{-RUR-U}}(t) := \text{rs}_k^S(t, 10) \times \text{SUF}_{10} \times U^{10\text{-RUR-U}}(t) \quad (20.1)$$

$$u_k^{W,10\text{-RUR-U}}(t) := \text{rs}_k^W(t, 10) \times \text{WUF}_{10} \times U^{10\text{-RUR-U}}(t) \quad (20.2)$$

We can now introduce the locational constraint. For a transmission line  $m$ , the constraint is a modification of (8) with additional terms based on unit-level reserve clearing, the 10-Min forward ramp on the load and renewable nodes, and the nodal allocation of the uncertainty cleared on the 10-Min RUR (Up) demand curve. This is given by,

$$\begin{aligned} -\mathcal{L}_m \leq & \sum_{i \in \mathcal{G}} \text{DF}_{i,m}^{\text{ax}} (g_i(t) + r_i^{10\text{-RUR-U}}(t)) \\ & + \sum_{j \in \mathcal{R}} \text{DF}_{j,m}^{\text{ax}} (g_j^S(t) + \Delta_j^S(t, 10) - u_j^{S,10\text{-RUR-U}}(t) \\ & \quad + g_j^W(t) + \Delta_j^W(t, 10) - u_j^{W,10\text{-RUR-U}}(t)) \\ & - \sum_{k \in \mathcal{L}} \text{DF}_{k,m}^{\text{ax}} (\ell_k(t) + \Delta_k^L(t, 10) + u_k^{L,10\text{-RUR-U}}(t)) \leq \mathcal{L}_m \end{aligned} \quad (21)$$

In (21), the negative signs in front of  $u^{S,10\text{-RUR-U}}$  and  $u^{W,10\text{-RUR-U}}$  and the positive signs in front of the  $u^{L,10\text{-RUR-U}}$  terms are due to the fact that we procure more up RUR to cover solar and wind over-forecasts and load under-forecasts.

Let  $\lambda_m^{10\text{-Up}}(t)$  denote the shadow price of the constraint in (21) for transmission line  $m$  at time  $t$ .

**10-Min RUR (Down).** The allocation of systemwide clearing for 10-Min RUR (Down) uncertainty  $U^{10\text{-RUR-D}}(t)$  to the load and renewable nodes is similar to the methodology described in (19), (20): for a node  $k$  at time  $t$ ,

$$u_k^{L,10\text{-RUR-D}}(t) := \text{rs}_k^L(t, 10) \times \text{LUF}_{10} \times U^{10\text{-RUR-D}}(t) \quad (22.1)$$

$$u_k^{S,10-RUR-D}(t) := rs_k^S(t, 10) \times SUF_{10} \times U^{10-RUR-D}(t) \quad (22.2)$$

$$u_k^{W,10-RUR-D}(t) := rs_k^W(t, 10) \times WUF_{10} \times U^{10-RUR-D}(t) \quad (22.3)$$

The locational constraint for 10-Min RUR (Down) is similar to the one for the up service given in (21) with the modification in signs to recognize that down RUR has an inverse relationship with the up RUR, which results in the following: for a transmission line  $m$ ,

$$\begin{aligned} -\mathcal{L}_m \leq & \sum_{i \in \mathcal{G}} DF_{i,m}^{\text{ax}} (g_i(t) - r_i^{10-RUR-D}(t)) \\ & + \sum_{j \in \mathcal{R}} DF_{j,m}^{\text{ax}} (g_j^S(t) + \Delta_j^S(t, 10) + u_j^{S,10-RUR-D}(t) \\ & \quad + g_j^W(t) + \Delta_j^W(t, 10) + u_j^{W,10-RUR-D}(t)) \\ & - \sum_{k \in \mathcal{L}} DF_{k,m}^{\text{ax}} (\ell_k(t) + \Delta_k^L(t, 10) - u_k^{L,10-RUR-U}(t)) \leq \mathcal{L}_m \end{aligned} \quad (23)$$

Let  $\lambda_m^{10-Down}(t)$  denote the shadow price of the constraint in (23) for transmission line  $m$  at time  $t$ .

**30-Min RUR.** As in the case of 10-Min RUR, we begin with the allocation of the uncertainty  $U^{30-RUR}(t)$  cleared on the 30-Min RUR demand curve to the load and renewable nodes. For a node  $k$  and a target time  $t$ , the 30-Min RUR uncertainty is assigned based on the nodal ramp share for the ramp occurring from 10 through 30 minutes following the target time:

$$u_k^{L,30-RUR}(t) := rs_k^L(t + 10, 20) \times LUF_{30} \times U^{30-RUR}(t) \quad (24.1)$$

$$u_k^{S,30-RUR}(t) := rs_k^S(t + 10, 20) \times SUF_{30} \times U^{30-RUR}(t) \quad (24.2)$$

$$u_k^{W,30-RUR}(t) := rs_k^W(t + 10, 20) \times WUF_{30} \times U^{30-RUR}(t) \quad (24.3)$$

The locational constraint for 30-Min RUR is a modification of (21) with additional terms for the 30-Min RUR products as well as the 30-Min forward ramp and uncertainties. For a transmission line  $m$  at time  $t$ ,

$$\begin{aligned} -\mathcal{L}_m \leq & \sum_{i \in \mathcal{G}} DF_{i,m}^{\text{ax}} (g_i(t) + r_i^{10-RUR-U}(t) + r_i^{30-RUR}(t)) \\ & + \sum_{j \in \mathcal{R}} DF_{j,m}^{\text{ax}} (g_j^S(t) + \Delta_j^S(t, 30) - u_j^{S,10-RUR-U}(t) - u_j^{S,30-RUR}(t) \\ & \quad + g_j^W(t) + \Delta_j^W(t, 30) - u_j^{W,10-RUR-U}(t) - u_j^{W,30-RUR}(t)) \\ & - \sum_{k \in \mathcal{L}} DF_{k,m}^{\text{ax}} (\ell_k(t) + \Delta_k^L(t, 30) + u_k^{L,10-RUR-U}(t) + u_k^{L,30-RUR}(t)) \leq \mathcal{L}_m \end{aligned} \quad (25)$$

Let  $\lambda_m^{30\text{-RUR}}(t)$  denote the shadow price of the constraint in (25) for transmission line  $m$  at time  $t$ .

**DAM modifications.** The constraints in (21), (23), (25) are modified to include the virtual INC, DEC, and UTC transactions. This is similar to the modifications made in (10).

## 7. Components of Congestion Prices for Energy and Reserves

Recall that the locational marginal price for energy is the sum of three components: (i) system marginal price, (ii) marginal cost of congestion, and (iii) marginal loss component (which is the product of system marginal price and the loss sensitivity factor multiplied by  $-1$ ).

The locational price of reserves will be the sum of (i) system marginal price of reserves, and (ii) marginal congestion cost of reserves. This document only discusses (ii), i.e., marginal cost of congestion. For the discussion on system marginal price of reserves, please see the RCSTF supplement on product nesting and resource level constraints.

Introducing nodal constraints for reserves implies that in addition to nodal price separation for reserves based on network congestion, there will be additional components in the congestion price for energy.

### 7.1. Nodal Congestion Prices for RUR Products

We begin with the 30-Min RUR products, since the 10-Min RUR products include the terms of the 30-Min RUR congestion prices.

**30-Min RUR.** The congestion price for the 30-Min RUR product at time  $t$  for node  $k$  is given by:

$$\lambda_k^{30\text{-RUR}}(t) = \sum_{m \in \mathcal{M}} \text{DF}_{k,m}^{\text{ax}} \lambda_m^{30\text{-RUR}}(t) \quad (26)$$

**10-Min RUR (Up).** The congestion price for the 10-Min RUR (Up) product at time  $t$  for node  $k$  is given by:

$$\lambda_k^{10\text{-Up}}(t) = \lambda_k^{30\text{-RUR}}(t) + \sum_{m \in \mathcal{M}} \text{DF}_{k,m}^{\text{ax}} \lambda_m^{10\text{-Up}}(t) \quad (27)$$

**10-Min RUR (Down).** The congestion price for the 10-Min RUR (Down) product at time  $t$  for node  $k$  is given by:

$$\lambda_k^{10\text{-Down}}(t) = - \sum_{m \in \mathcal{M}} \text{DF}_{k,m}^{\text{ax}} \lambda_m^{10\text{-Down}}(t) \quad (28)$$

### 7.2. Nodal Congestion Prices for Contingency Reserves

Similar to RUR, we will begin with the 30-Min secondary reserve products.

**30-Min SecR.** The congestion price for the 30-Min SecR product at time  $t$  for node  $k$  is given by:

$$\lambda_k^{\text{SecR}}(t) = \sum_{m \in \mathcal{M}} \text{DF}_{k,m}^{\text{ax}} \left( \lambda_m^{\text{SecR}^a}(t) + \lambda_m^{\text{SecR}^b}(t) \right) \quad (29)$$

**SR.** The congestion price for the SR product at time  $t$  for node  $k$  is given by:

$$\lambda_k^{\text{SR}}(t) = \lambda_k^{\text{SecR}}(t) + \sum_{m \in \mathcal{M}} \text{DF}_{k,m}^{\text{ax}} \left( \lambda_m^{\text{SR}^a}(t) + \lambda_m^{\text{SR}^b}(t) \right) \quad (30)$$

### 7.3. Nodal Congestion Price of Energy

Finally, the congestion price for energy will involve components based on the shadow prices of the different locational constraints on reserves. The cumulative marginal congestion price of energy at time  $t$  for node  $k$  is given by,

$$\lambda_k^e(t) = \lambda_k^{\text{SR}}(t) + \lambda_k^{10\text{-Up}}(t) - \lambda_k^{10\text{-Down}}(t) + \sum_{m \in \mathcal{M}} \text{DF}_{k,m}^{\text{ax}} \lambda_m^e(t) \quad (31)$$