Electrical Theory

Math Review

PJM State & Member Training Dept.

Objectives

By the end of this presentation the Learner should be able to:

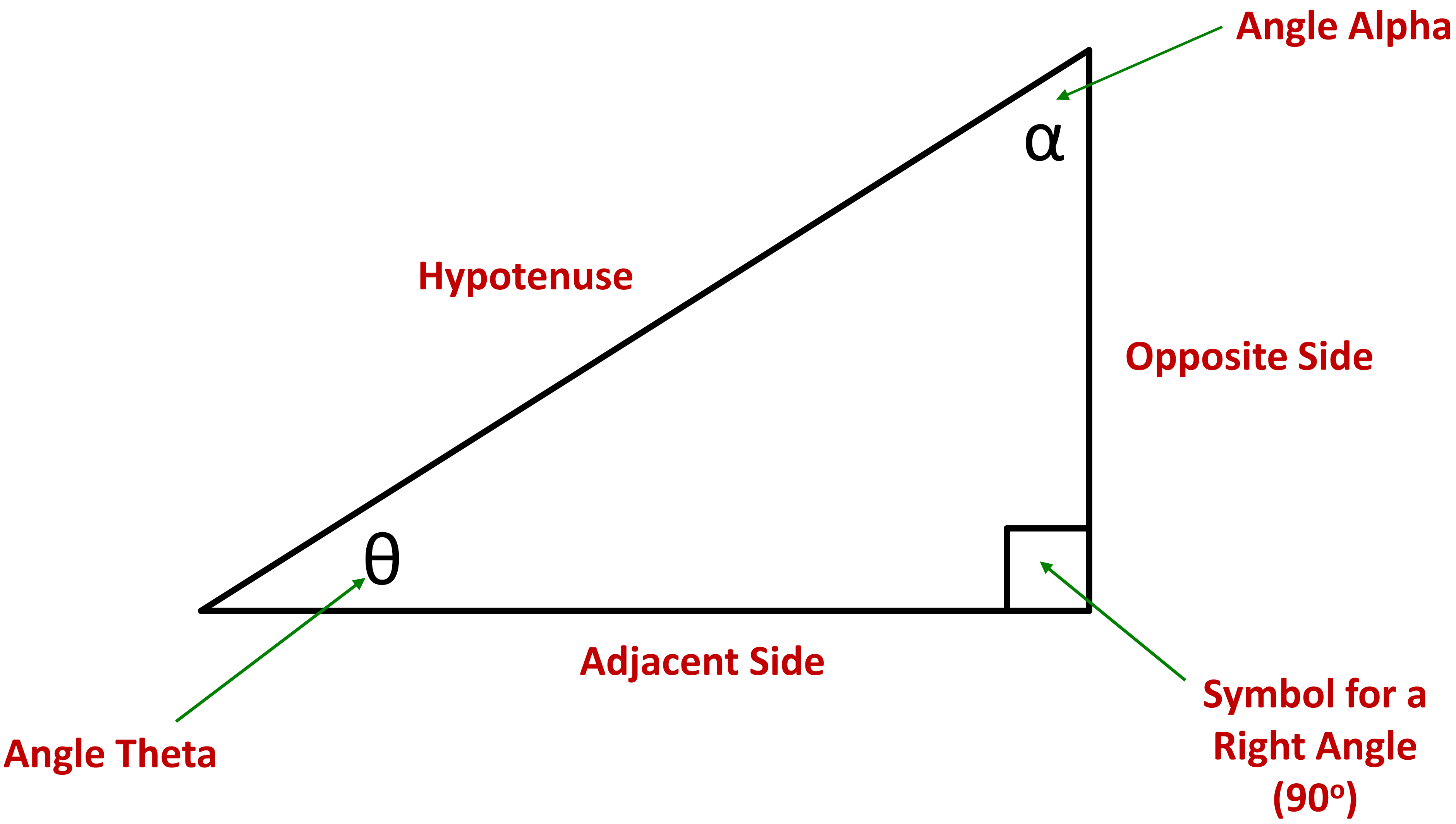
- Use the basics of trigonometry to calculate the different components of a right triangle
- Compute Per-Unit Quantities
- Identify the two components of Vectors

Right Triangles

Mathematics Review

- To be able to understand basic AC power concepts, a familiarization with the relationships between the angles and sides of a right triangle is essential
- A right triangle is defined as a triangle in which one of the three angles is equal to 90^o

Mathematics Review



Mathematics Review

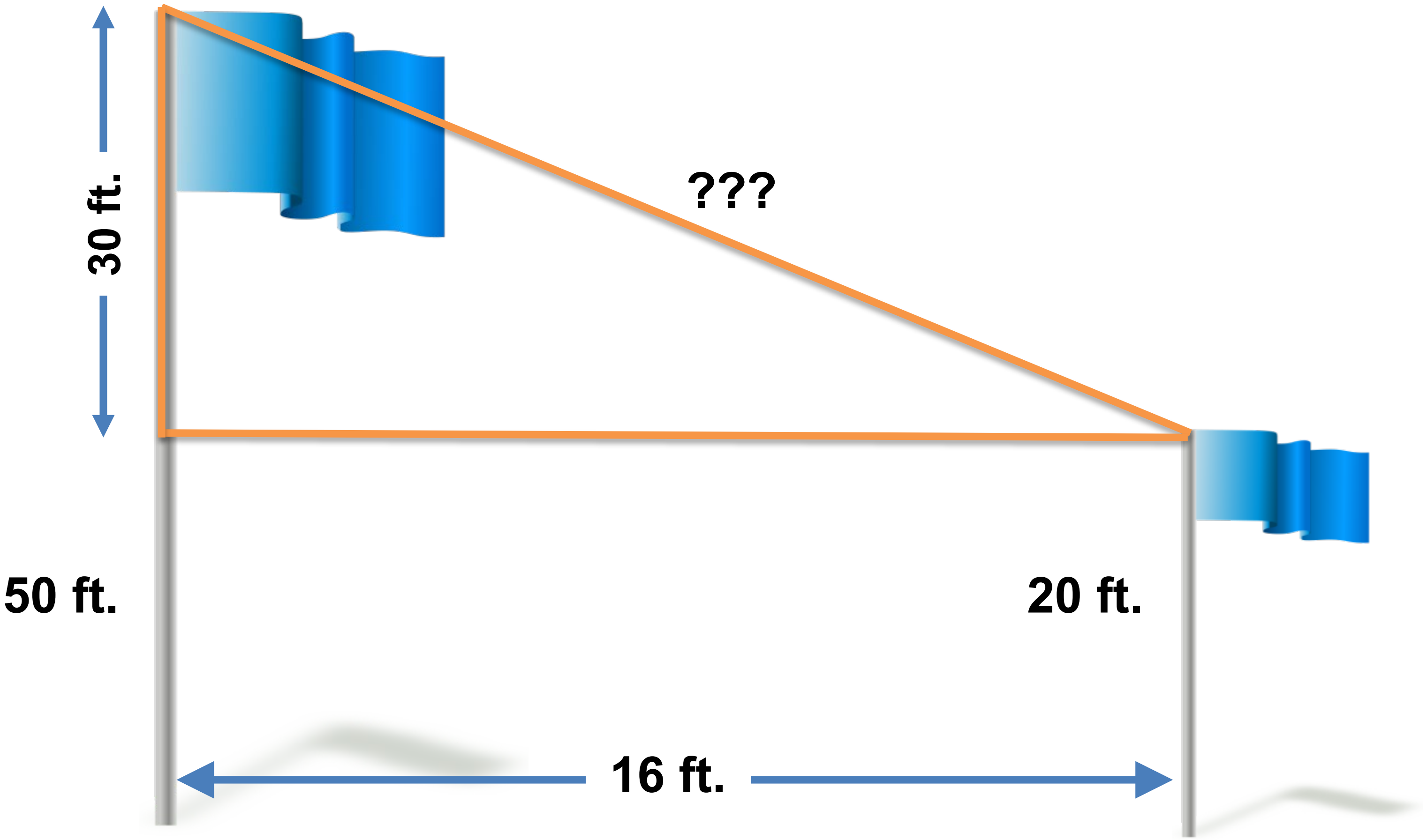
- Given the lengths of two sides of a right triangle, the third side can be determined using the Pythagorean Theorem

$$\text{Hypotenuse}^2 = \text{Opposite}^2 + \text{Adjacent}^2$$

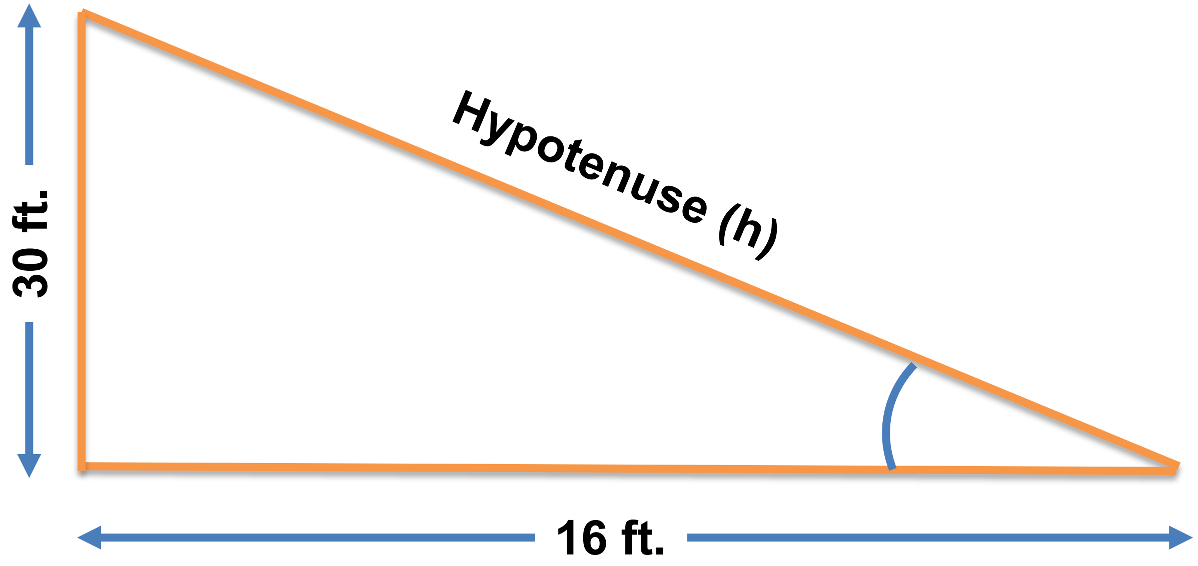
Example:

A rope stretches from the top of one pole 50 feet high to the top of another pole 20 feet high, standing 16 feet away.

How long is the rope?



Example:



$$h^2 = a^2 + o^2$$
$$h^2 = 16^2 + 30^2$$
$$h^2 = 256 + 900$$
$$h^2 = 1156$$
$$h = \sqrt{1156}$$
$$h = 34$$

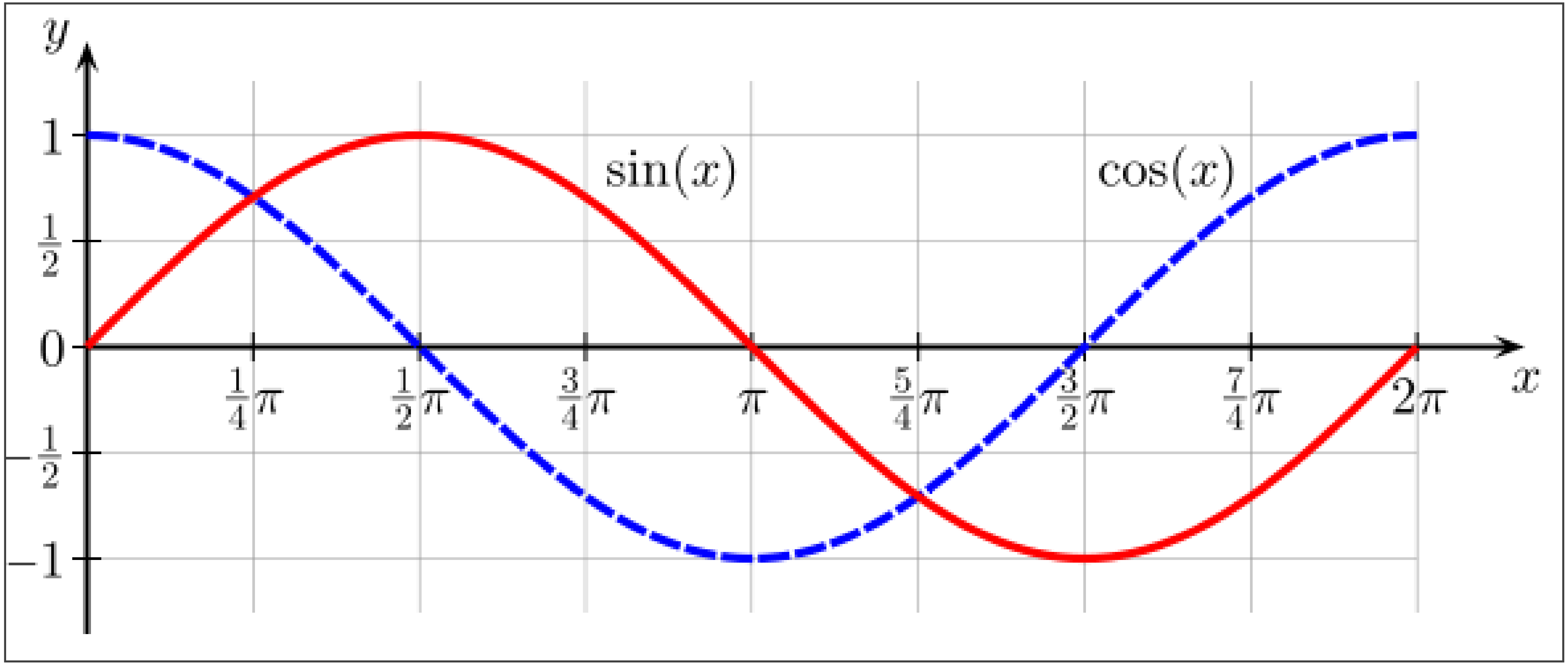
Mathematics Review

- Once the sides are known, the next step in solving the right triangle is to determine the two unknown angles of the right triangle
- All of the angles of any triangle always add up to 180^o
- In solving a right triangle, the remaining two unknown angles must add up to 90^o
- Basic trigonometric functions are needed to solve for the values of the unknown angles

Trigonometry

Mathematics Review

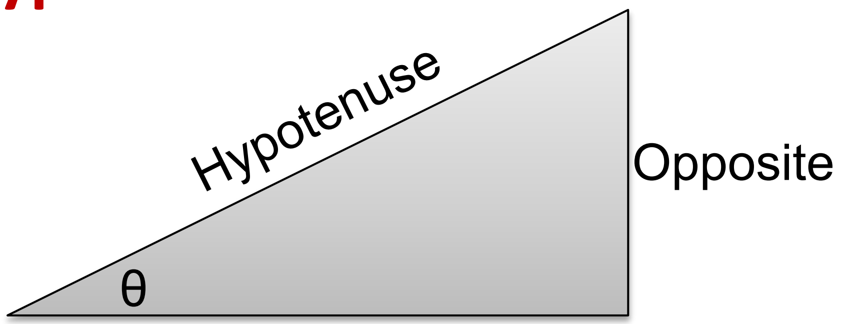
- The sine function is a periodic function in that it continually repeats itself



Mathematics Review

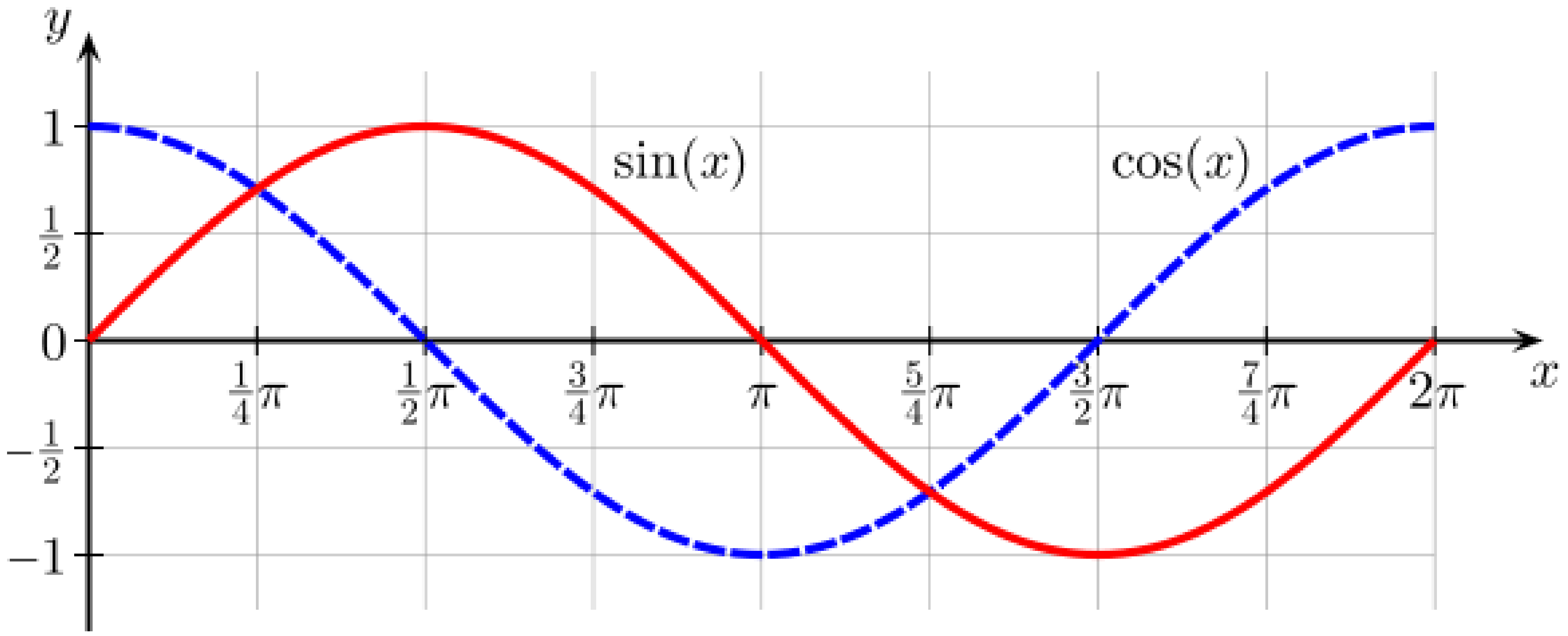
- In order to solve right triangles, it is necessary to know the value of the sine function between 0^o and 90^o
- Sine of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the hypotenuse

SIN θ = Opposite Side / Hypotenuse



Mathematics Review

- Cosine function is a periodic function that is identical to the sine function except that it leads the sine function by 90^o



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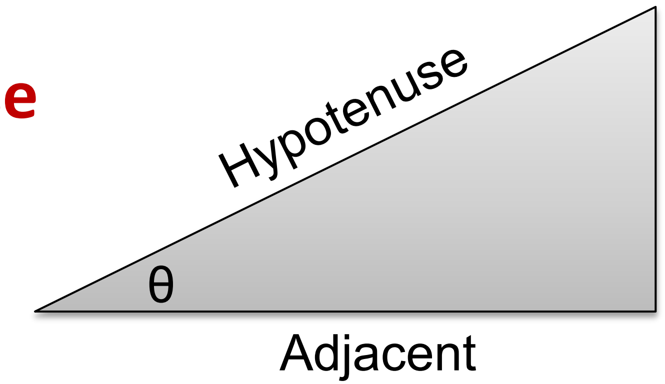
Mathematics Review

- As an example, the cosine function at 0^o is 1 whereas the sine function does not reach the value of 1 until 90^o
- Cosine function of either of the unknown angles of a right triangle is the ratio of the length of the adjacent side to the length of the hypotenuse

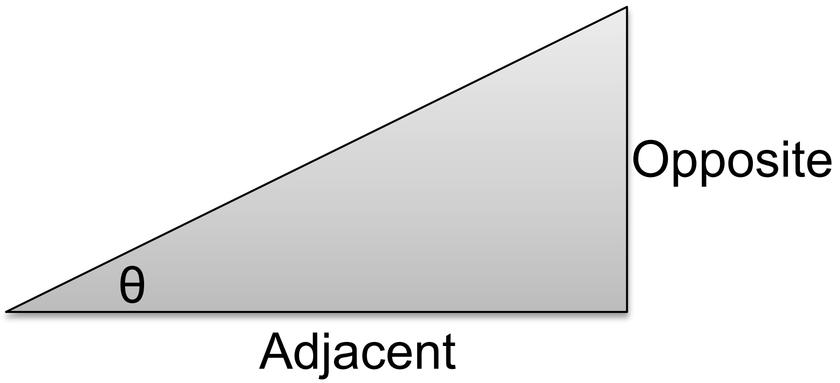
COS θ = Adjacent Side / Hypotenuse



Mathematics Review

- The tangent function of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the adjacent side

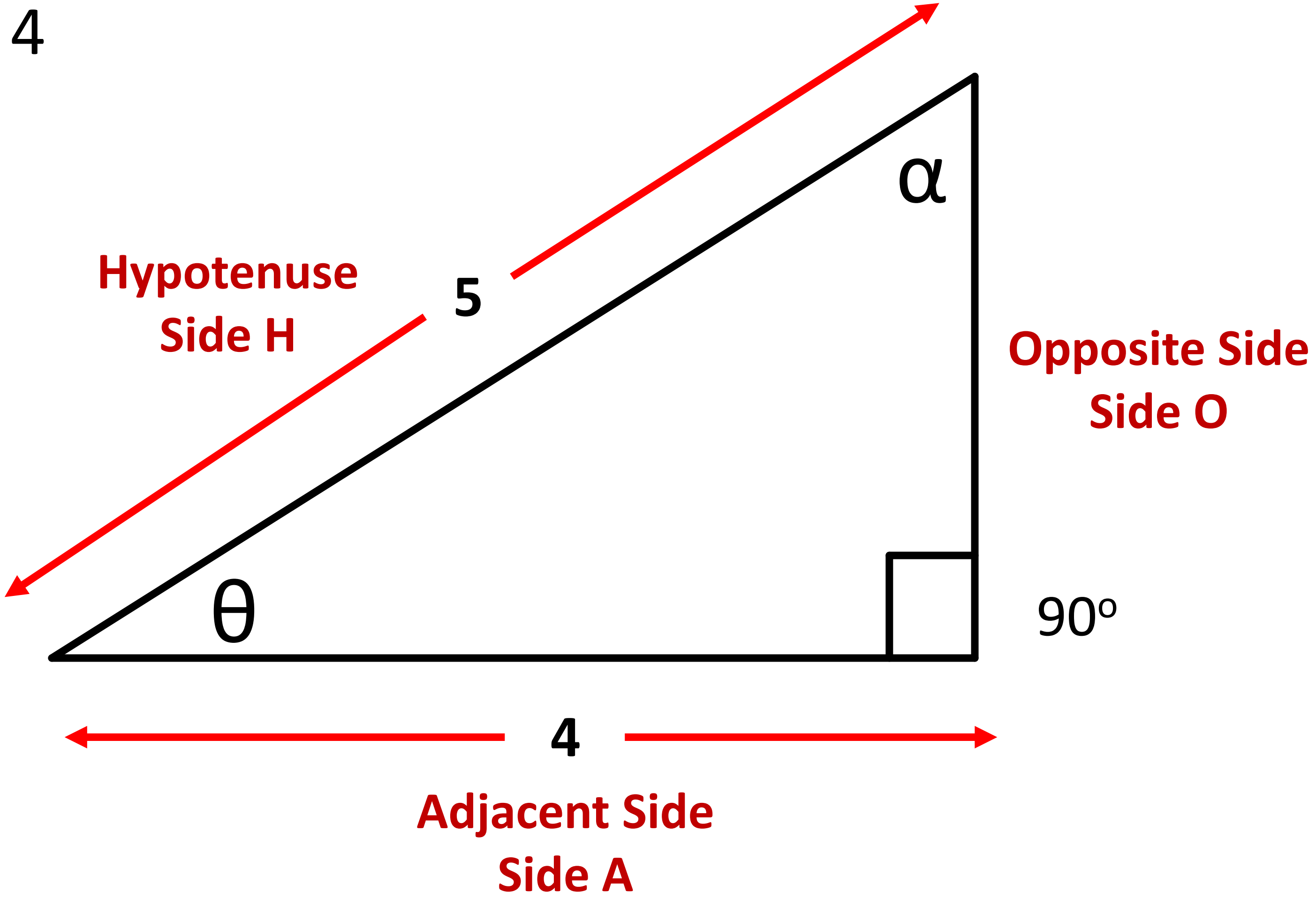
TAN θ = Opposite Side / Adjacent Side



Mathematics Review

Example:

- Given: Side H = 5, Side A = 4
- Find: Side O, Angle θ and Angle α



Mathematics Review

- Find Side O:

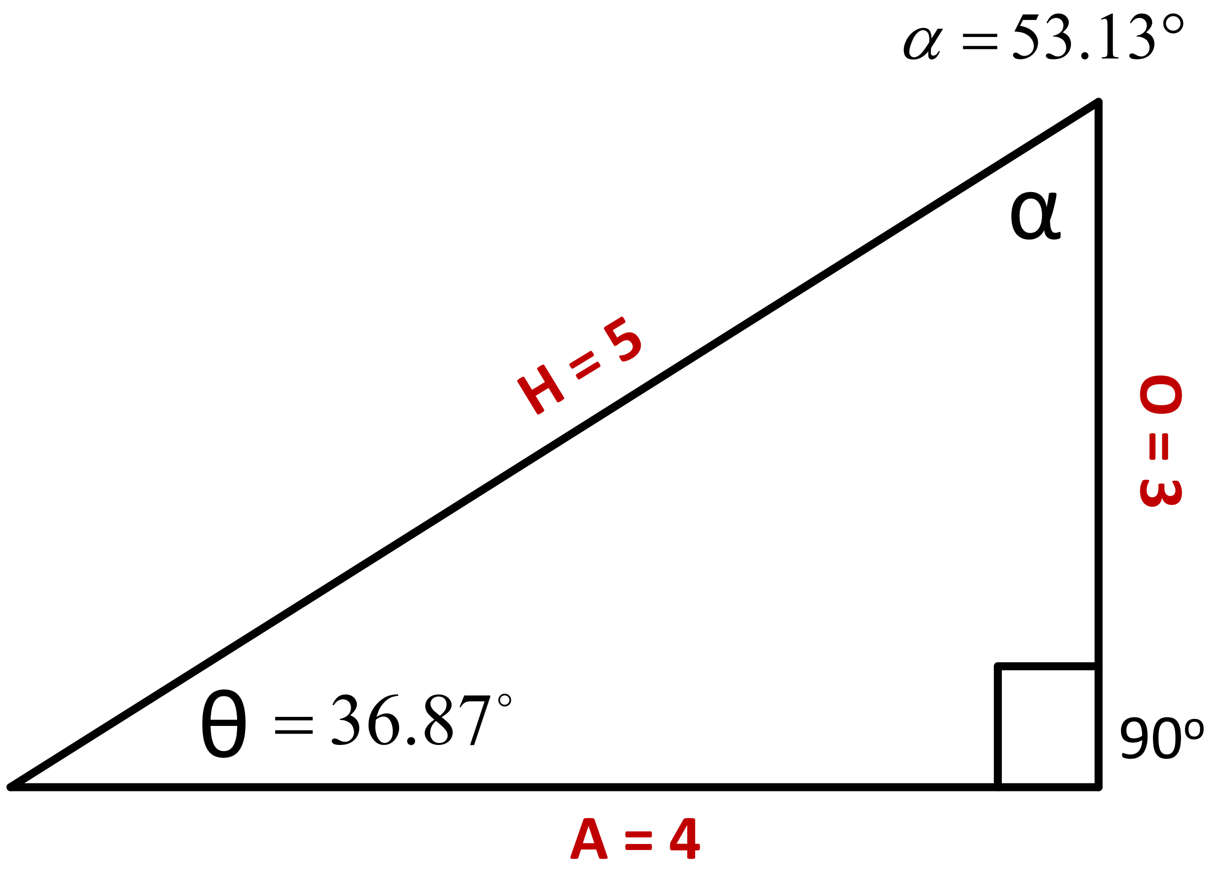
$$H^2 = A^2 + O^2$$
$$25 = 16 + O^2$$
$$25 - 16 = O^2$$
$$\sqrt{25-16} = O$$
$$O = \sqrt{9} = 3$$

- Find θ:

$$\cos\theta = \frac{A}{H} = \frac{4}{5} = .8$$
$$\cos^{-1}(.8) = 36.87^\circ$$

- Find α:

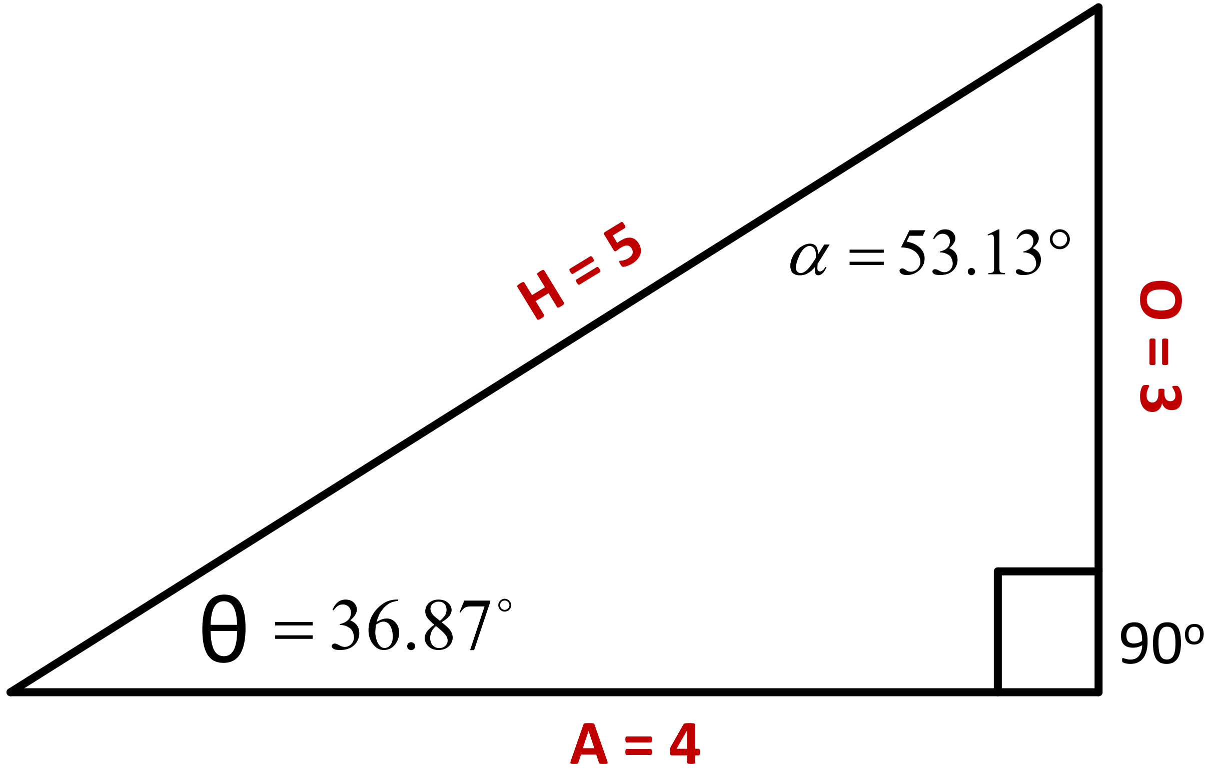
$$180° - 90° - 36.87° = 53.13°$$



Mathematics Review

$$\left.\begin{aligned}\theta &= \sin^{-1}\left(\frac{3}{5}\right)\\ \theta &= \cos^{-1}\left(\frac{4}{5}\right)\\ \theta &= \tan^{-1}\left(\frac{3}{4}\right)\end{aligned}\right\} 36.87$$

$$\left.\begin{aligned}\alpha &= \sin^{-1}\left(\frac{4}{5}\right)\\ \alpha &= \cos^{-1}\left(\frac{3}{5}\right)\\ \alpha &= \tan^{-1}\left(\frac{4}{3}\right)\end{aligned}\right\} 53.13$$



Per Unit Quantities

Mathematics Review

- Ratios play an important part in estimating power system performance
  - Relationship between two quantities as a fraction
  - Used when the relationship of two pairs of values is the same, and one of two similarly related values is known
    - Example: if you can drive 120 miles in 2 hours, how many miles could you drive in 8 hours?

$$\frac{120\ miles}{2\ hours} = \frac{X\ miles}{8\ hours} = \frac{120\ miles(8\ \cancel{hours})}{2\ \cancel{hours}} = \frac{X\ miles\ (8\ \cancel{hours})}{\cancel{8\ hours}} = \quad 480\ miles$$

Mathematics Review

- Quantities on the power system are often specified as a percentage or a per-unit of their base or nominal value
  - Makes it easier to see where a system value is in respect to its base value
  - How it compares between different parts of the system with different base values
  - Allow for a dispatcher to view the system and quickly obtain a feel for the voltage profile

Mathematics Review

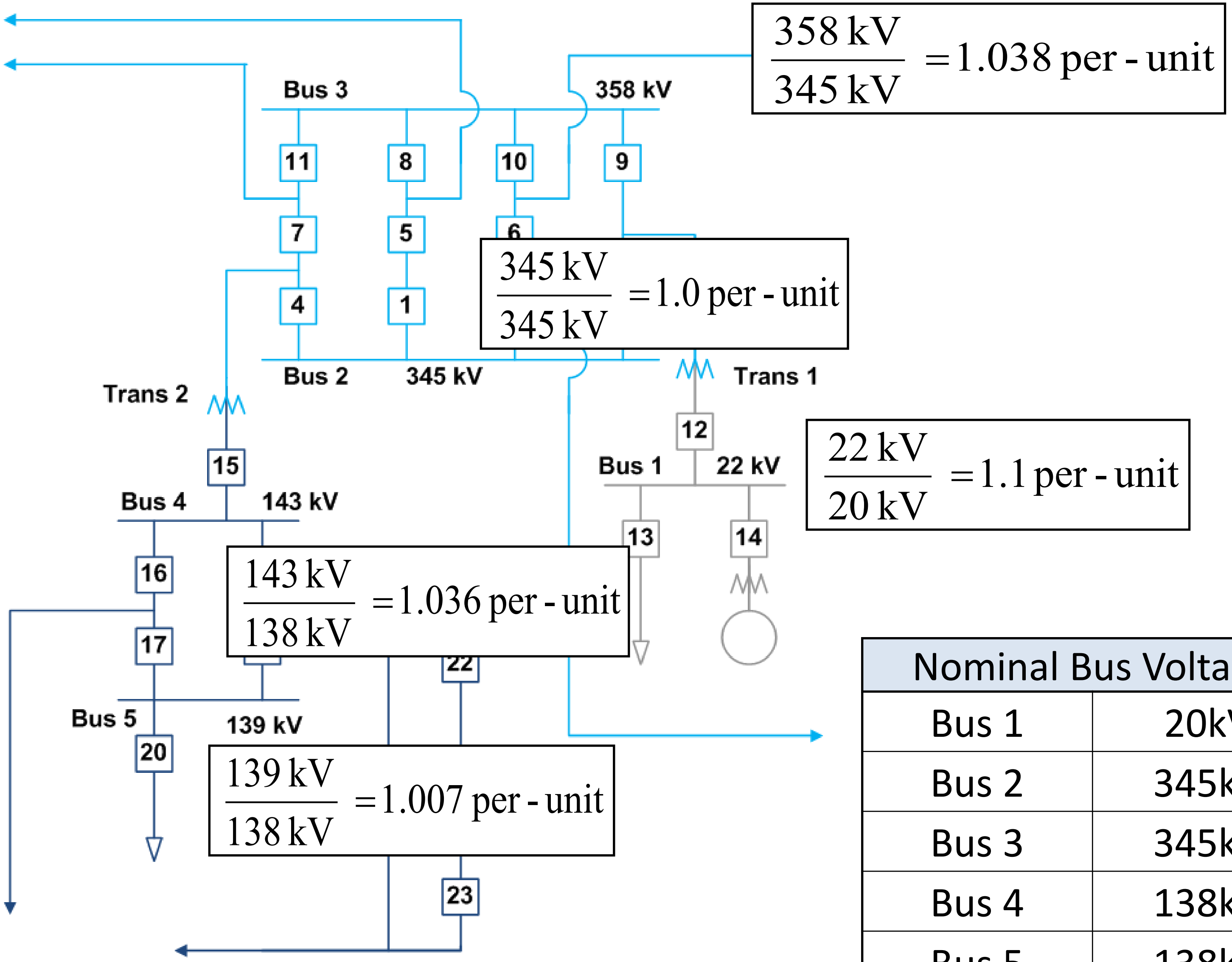
- Assume that, at a certain substation, the voltage being measured is 510 kV on the 500 kV system. What is its per-unit value with respect to the nominal voltage?

Base or nominal voltage = 500 kV

Measured voltage = 510 kV

510 kV / 500 kV = <u>1.02 per-unit</u> or 102%

Mathematics Review



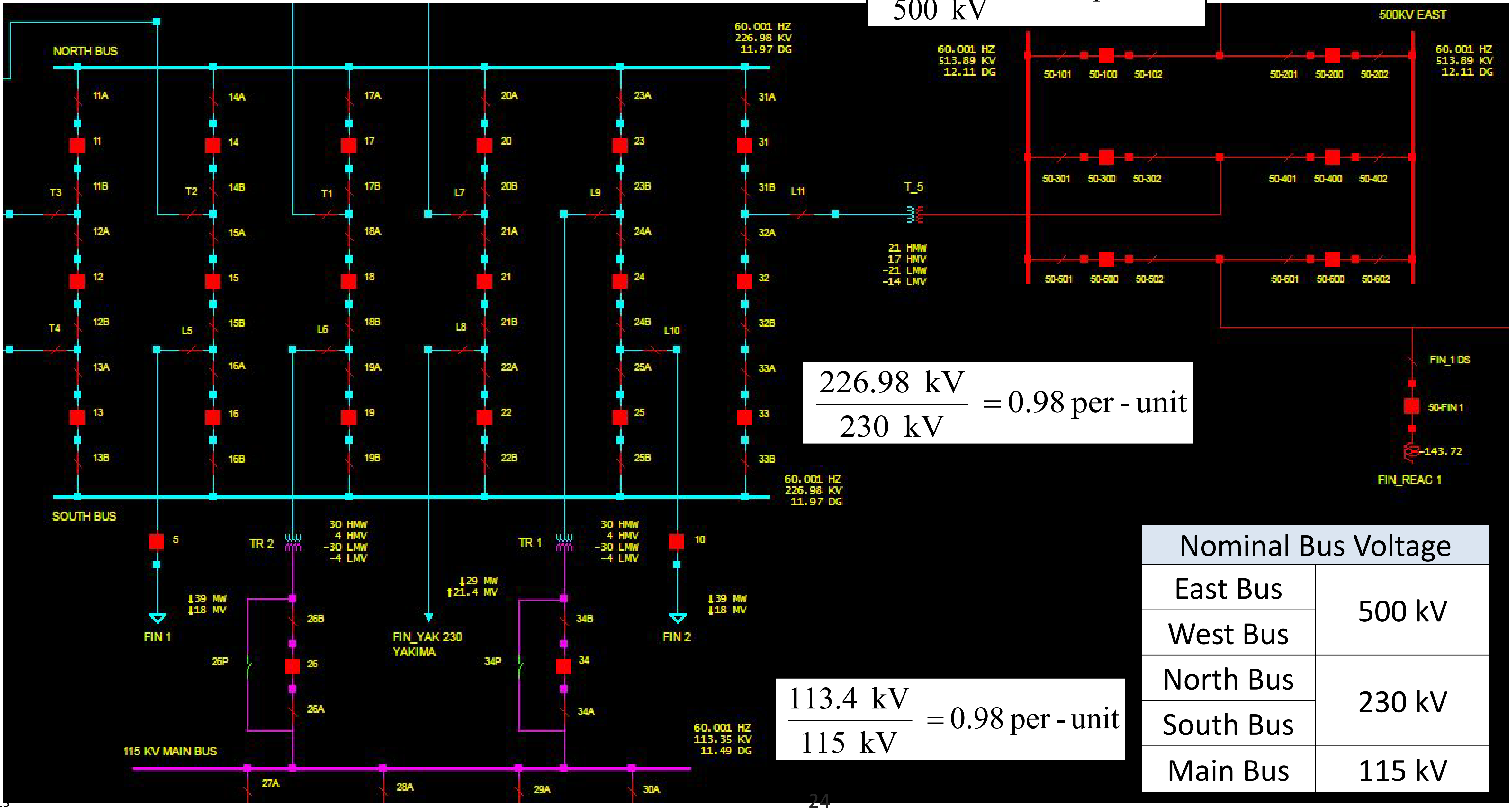
Nominal Bus Voltage	
Bus 1	20kV
Bus 2	345kV
Bus 3	345kV
Bus 4	138kV
Bus 5	138kV

Mathematics Review

$$\frac{513.9\ \text{kV}}{500\ \text{kV}} = 1.03\ \text{per-unit}$$

$$\frac{226.98\ \text{kV}}{230\ \text{kV}} = 0.98\ \text{per-unit}$$

$$\frac{113.4\ \text{kV}}{115\ \text{kV}} = 0.98\ \text{per-unit}$$



Nominal Bus Voltage	
East Bus	500 kV
West Bus	
North Bus	230 kV
South Bus	
Main Bus	115 kV

Answer Question 4 & 5

Question 4

- Assume that the loss of a 1000 MW generating unit will typically result in a 0.2 Hz dip in system frequency
- Estimate the frequency dip for the loss of an 800 MW generating unit

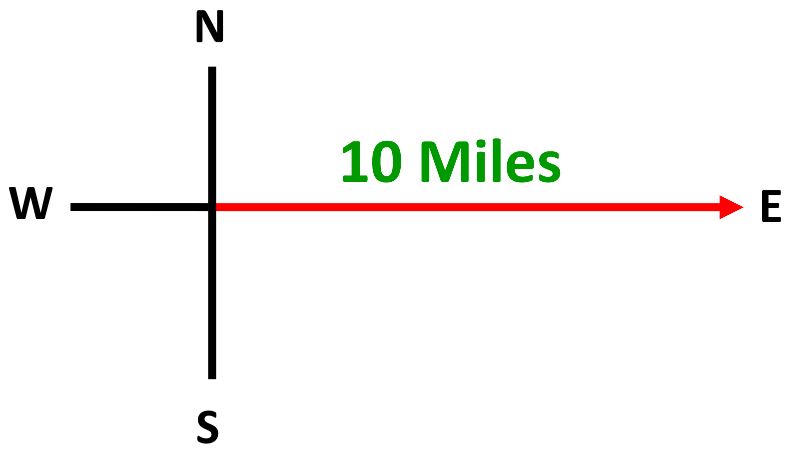
Question

- Assume that the loss of a 500 MW generating unit will typically result in a 0.3 Hz dip in system frequency.
- Estimate the frequency dip for the loss of an 300 MW generating unit.

Vectors

Vectors

- A vector is alternative way to represent a sinusoidal function with amplitude, and phase information
- Vectors are usually written in bold with an arrow over the top of the letter, ($\vec{\boldsymbol{A}}$)
- A vector's length represents **magnitude**
- A vector's **direction** represents the phase angle
  - Example: 10 miles east



Vectors

- Horizontal line to the right is positive; horizontal line to the left is negative
- Vertical line going up is positive; vertical line going down is negative
- Arrowhead on the end away from the point of origin indicates the direction of the vector and is called the displacement vector
- Vectors can go in any direction in space

Vectors

- The difference between a scalar quantity and a vector:
  a) A scalar quantity is one that can be described with a single number, including any units, giving its size or magnitude
  b) A vector quantity is one that deals inherently with both magnitude and direction

Conceptual Question 6

- There are places where the temperature is $+20^{o}$ C at one time of the year and -20^{o} C at another time.
  - Do the plus and minus signs that signify positive and negative temperatures imply that temperature is a vector quantity?

Question 7

Which of the following statements, if any, involves a vector?

a) I walked two miles along the beach

b) I walked two miles due north along the beach

c) A ball fell off a cliff and hit the water traveling at 17 miles per hour

d) A ball fell off a cliff and traveled straight down 200 feet

e) My bank account shows a negative balance of -25 dollars

Vectors

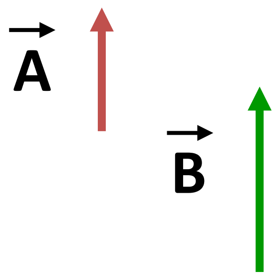
- When adding vectors, the process must take into account both the magnitude and direction of the vectors
- When adding two vectors, there is always a resultant vector, R, and the addition is written as follows:

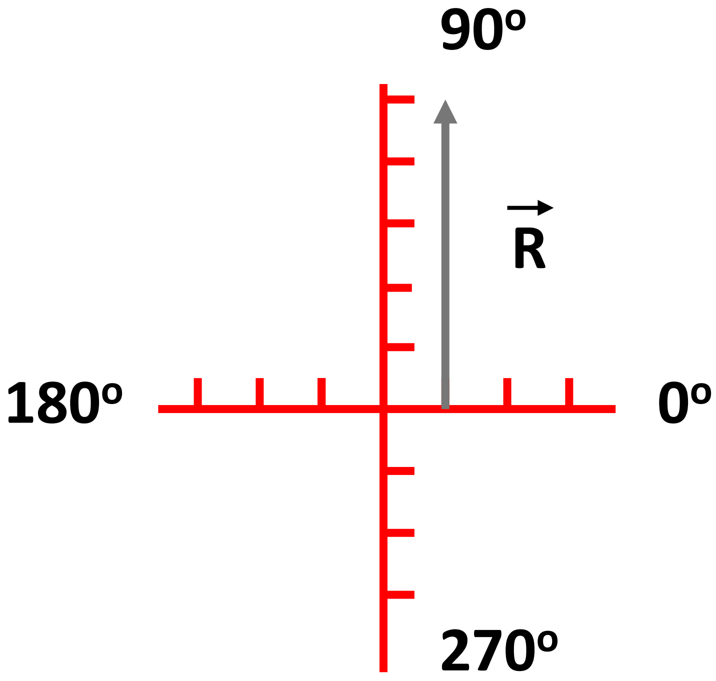
$$\vec{\boldsymbol{R}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}}$$

Vectors

Example: Adding vectors in the same direction

- Vector $\vec{A}$ has a length of 2 and a direction of 90^o
- Vector $\vec{B}$ has a length of 3 and a direction of 90^o

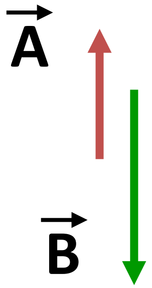


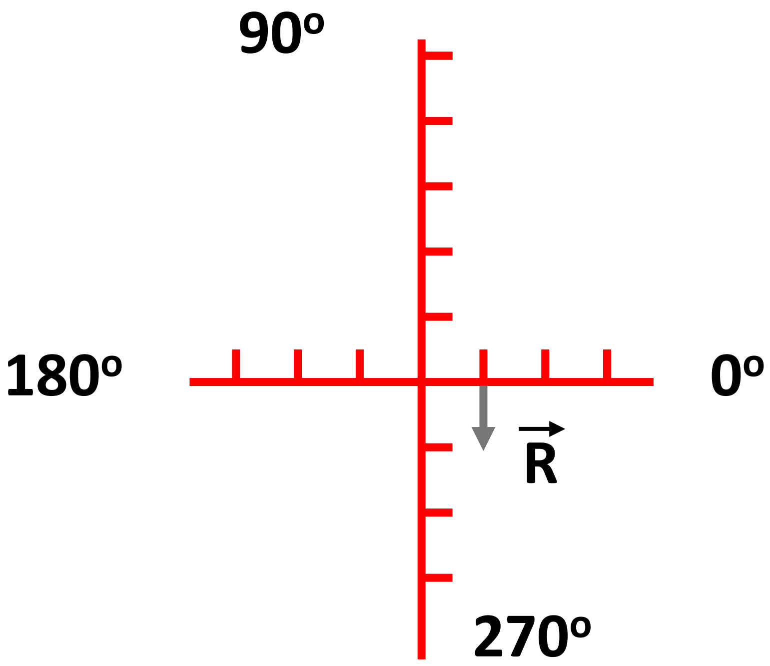


Vectors

Example: Adding vectors in the opposite direction

- Vector $\vec{A}$ has a length of 2 and a direction of 90º
- Vector $\vec{B}$ has a length of 3 and a direction of 270^0





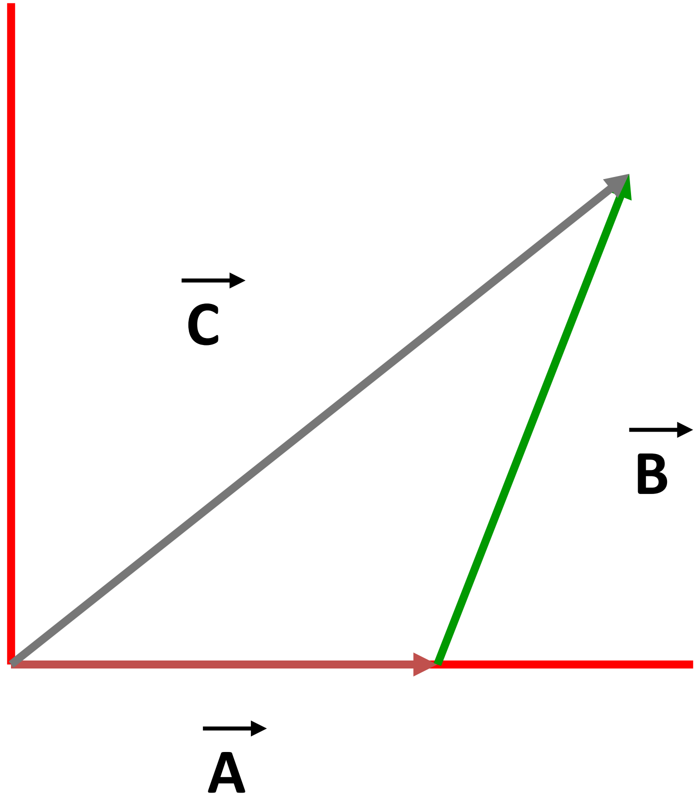
Vectors

- Subtraction of one vector from another is carried out in a way that depends on the following:

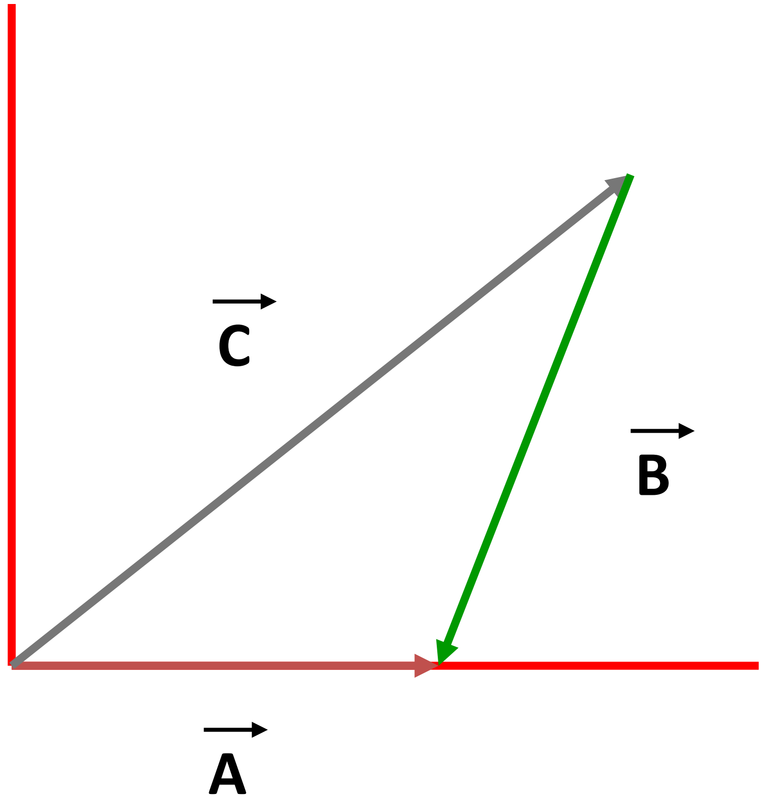
When a vector is multiplied by -1, the magnitude of the vector remains the same, but the direction of the vector is reversed

- Vector subtraction is carried out exactly like vector addition except that one of the vectors added is multiplied by the scalar factor of -1

Vectors



$$\vec{C} = \vec{A} + \vec{B}$$



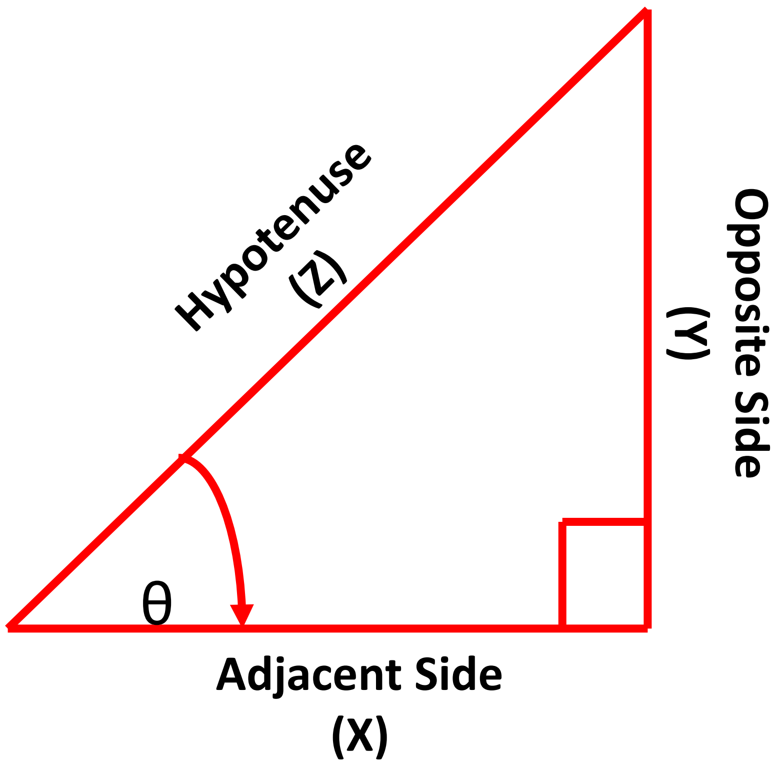
$$\vec{A} = \vec{C} + (-\vec{B})$$

Vectors

- If the magnitude and direction of a vector is known, it is possible to find the components of the vector
- The process is called “resolving the vector into its components”
- If the vector components are perpendicular and form a right triangle, the process can be carried out with the aid of trigonometry

Vectors

- To calculate the sum of two or more vectors using their components (x and y) in the vertical and horizontal directions, trigonometry is used



Vectors

- The Pythagorean Theorem is a special relationship that exists in any triangle and describes the relationship between the lengths of the sides of a right triangle

$$z^2 = x^2 + y^2$$

- Three basic trigonometric functions defined by a right triangle are:

sin θ = y/z = opposite side/hypotenuse

cos θ = x/z = adjacent side/hypotenuse

tan θ = y/x = opposite side/adjacent side

tan θ = sin θ/cos θ

Vectors

- To find theta, the inverse of the trigonometric function must be used

$$\theta = \sin^{-1} y/z$$

$$\theta = \cos^{-1} x/z$$

$$\theta = \tan^{-1} y/x$$

- When adding vectors, magnitude is found by:

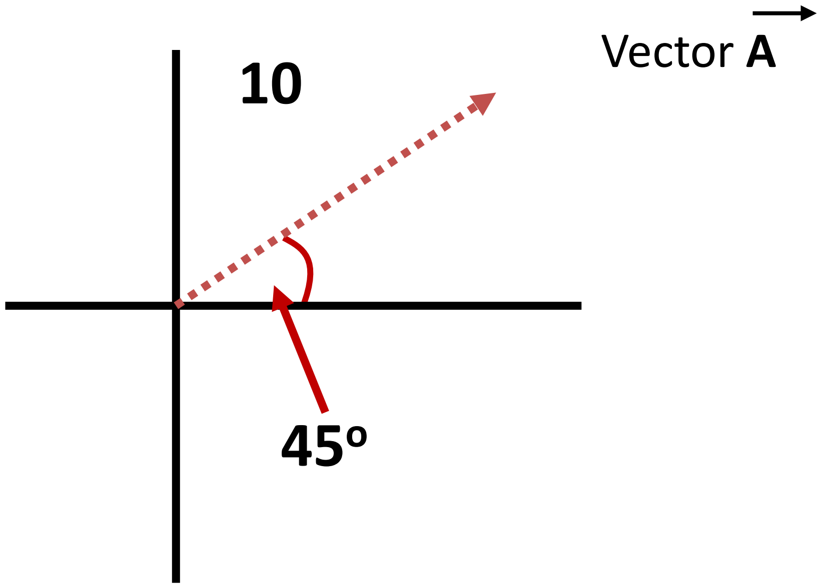
$$\vec{R} = \sqrt{\left(\overrightarrow{R_x}\right)^2 + \left(\overrightarrow{R_y}\right)^2}$$

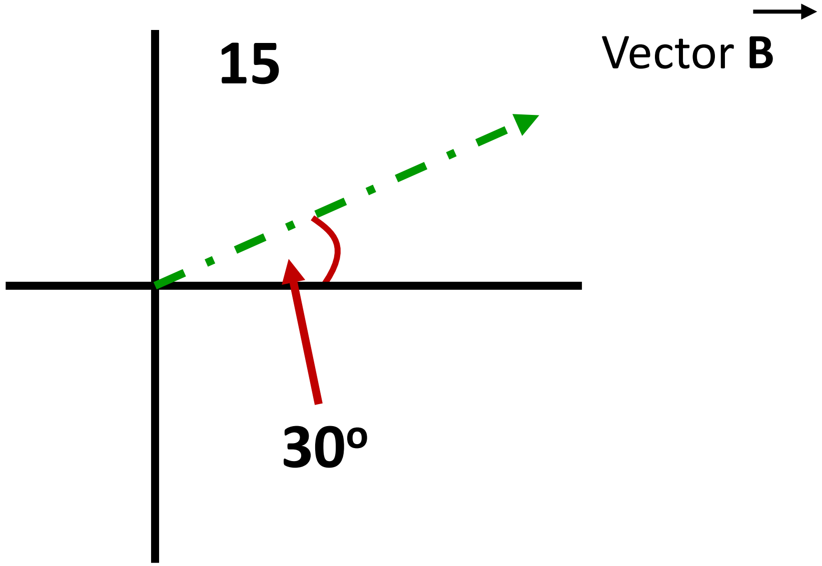
- The direction of the resultant, R, is found by:

$$\theta = \sin^{-1} (R_y/R)$$

Vectors

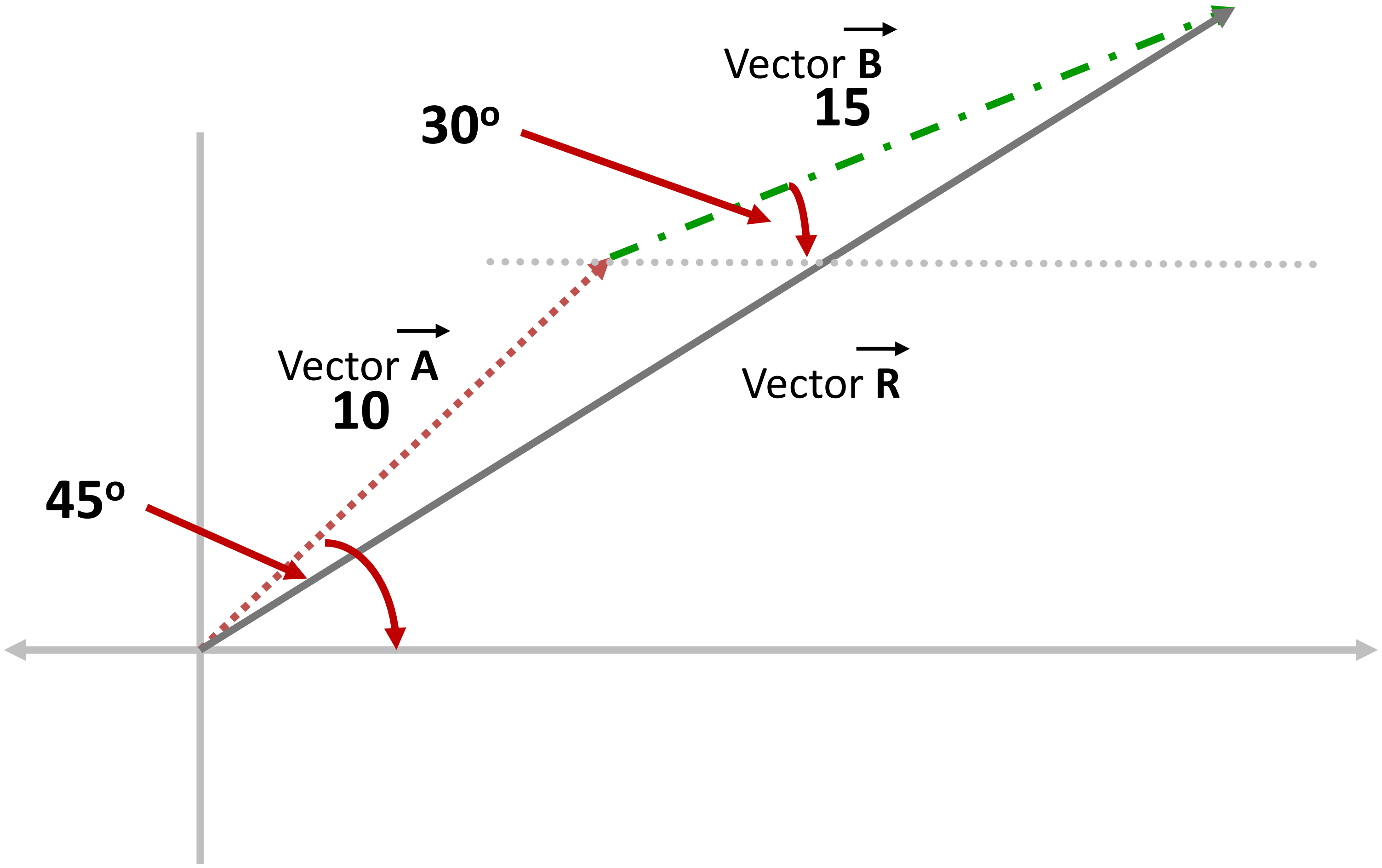
- Vectors can be added either in the same direction or in opposite directions



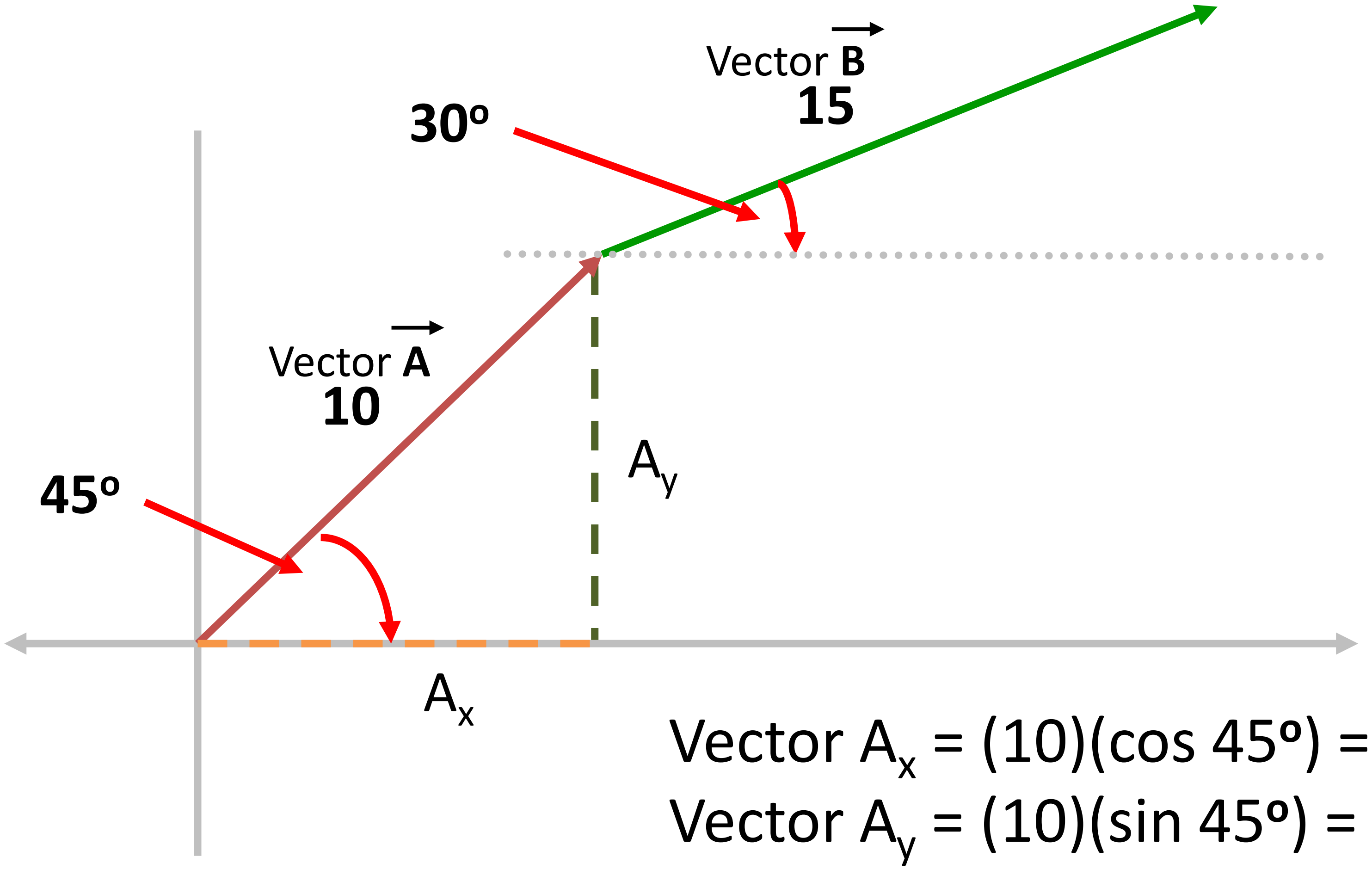


Vectors

Adding vectors:



Vectors



Vector A_x = (10)(cos 45º) = 7.07

Vector A_y = (10)(sin 45º) = 7.07

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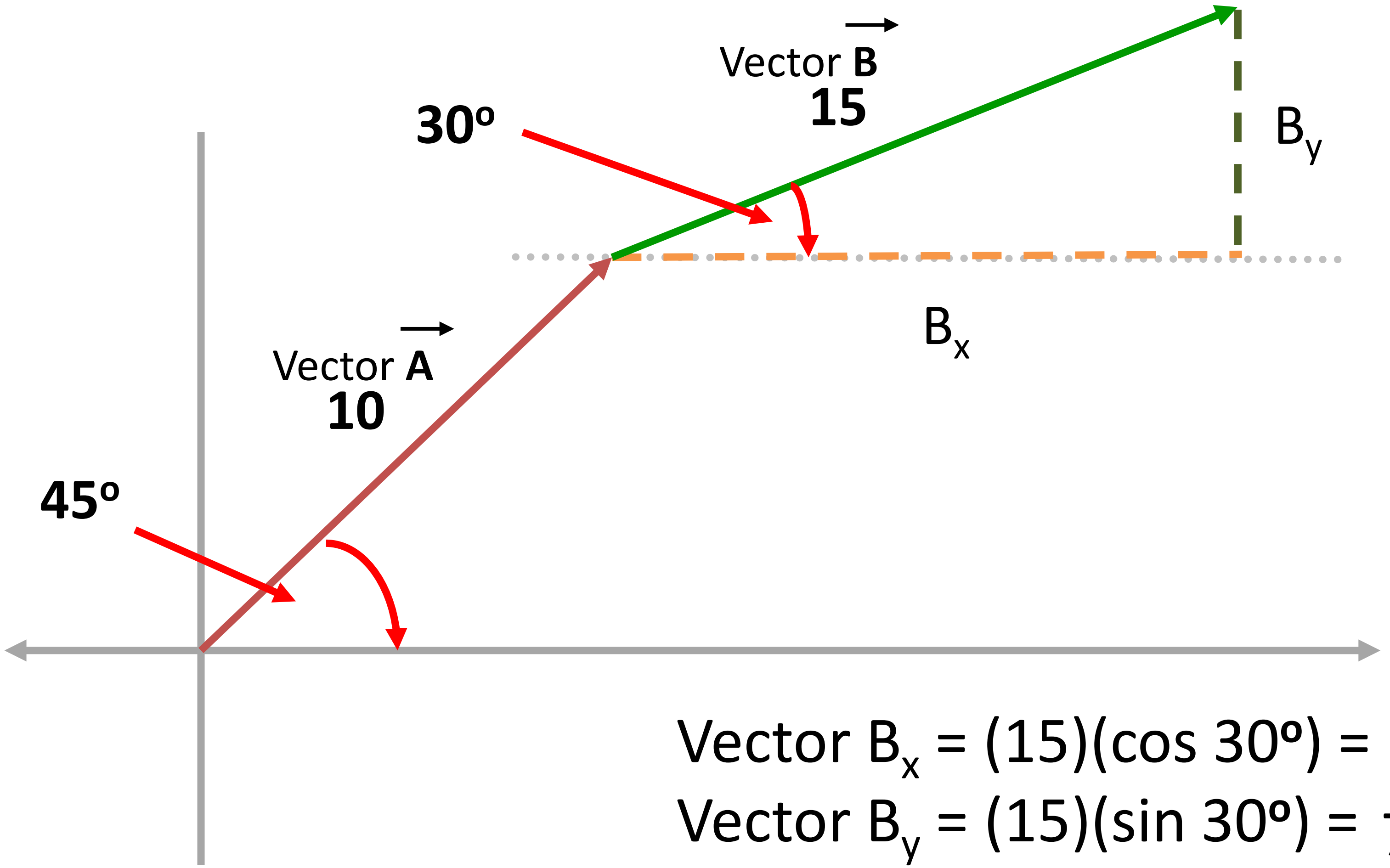
Vectors



Vector B_x = (15)(cos 30º) = 12.99

Vector B_y = (15)(sin 30º) = 7.5

Vectors

Determine the resulting vectors, R_x and R_y:

$\overrightarrow{R_X} = \overrightarrow{A_X} + \overrightarrow{B_X}$

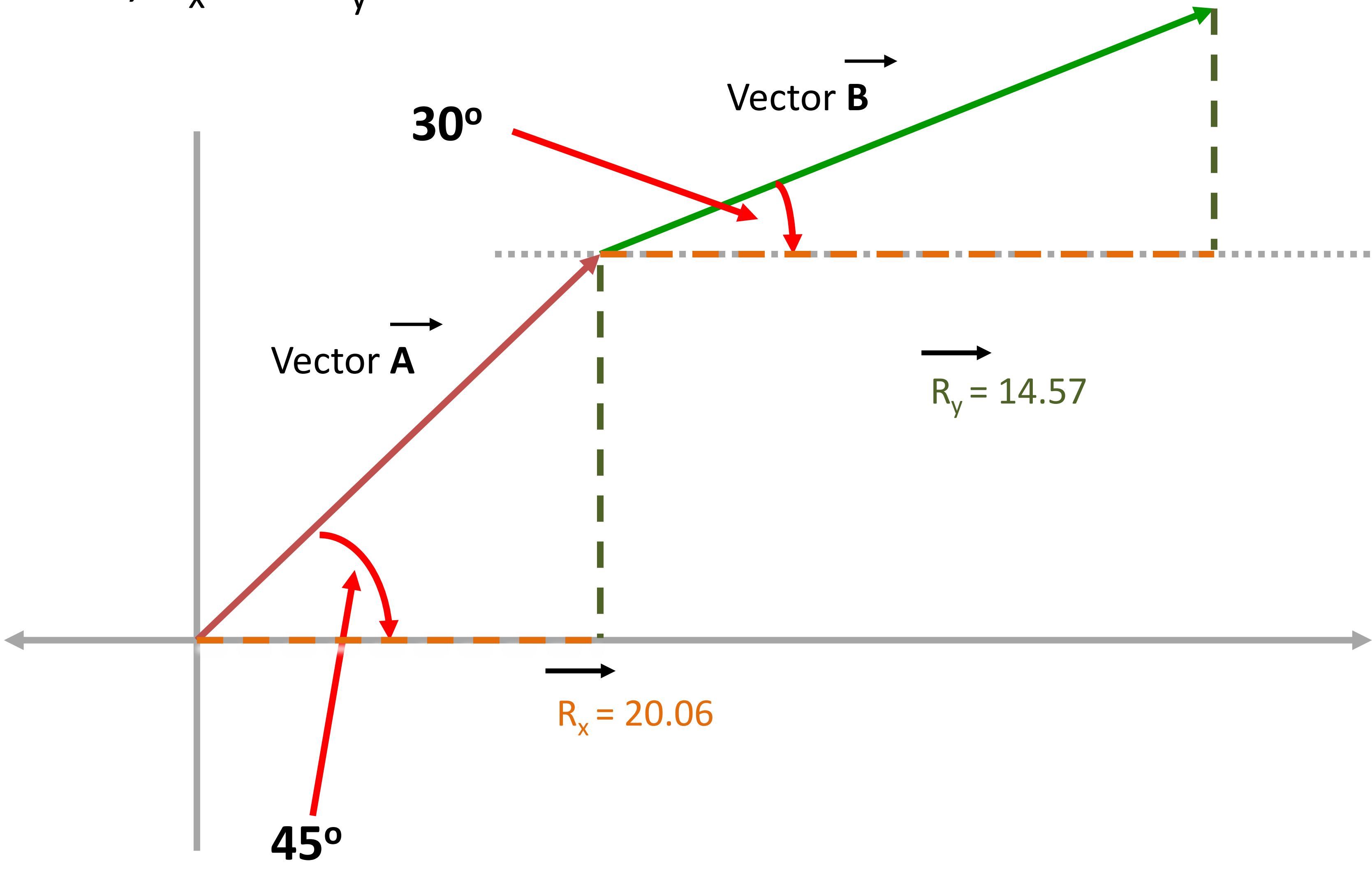
$\overrightarrow{R_X} = 7.07 + 12.99$

$\overrightarrow{R_X} = 20.06$

$\overrightarrow{R_y} = \overrightarrow{A_y} + \overrightarrow{B_y}$

$\overrightarrow{R_y} = 7.07 + 7.5$

$\overrightarrow{R_y} = 14.57$



Vectors

$$\vec{R} = \sqrt{\vec{R}_x^2 + \vec{R}_y^2}$$

$$\vec{R} = \sqrt{20.06^2 + 14.57^2}$$

$$\vec{R} = \sqrt{614.7}$$

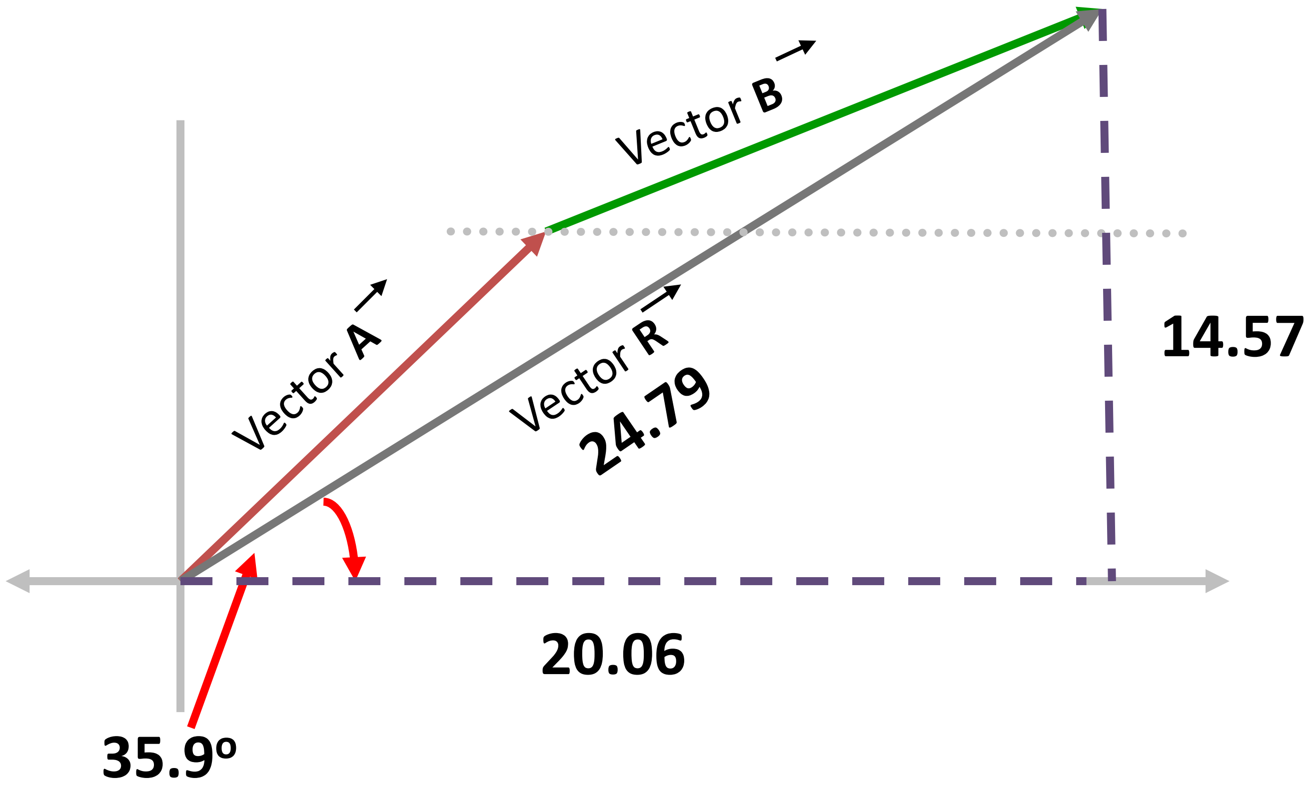
$$\vec{R} = 24.79$$

$$\theta = \sin^{-1}\left(\frac{R_y}{R}\right)$$

$$\theta = \sin^{-1}\left(\frac{14.57}{24.79}\right)$$

$$\theta = \sin^{-1}(.587)$$

$$\theta = 35.9^\circ$$



Vectors

- Polar notation expresses a vector in terms of both a magnitude and a direction, such as:

$$\mathbf{M} \angle {}^{\mathbf{o}}$$

where:

M is the magnitude of the vector

o is the direction in degrees

Example:

Vector with a magnitude of 10 and a direction of -40 degrees $10 \angle -40^{o}$

Vectors

- Multiplication in polar notation:

Multiply the magnitudes/add the angles

$$(50 \angle 25^{o}) + (25 \angle 30^{o}) = 1250 \angle 55^{o}$$

- Division in polar notation:

Divide the magnitudes/subtract the angles

$$\frac{(50 \angle 25^{o})}{(25 \angle 30^{o})} = 2 \angle -5^{o}$$

Questions?

PJM Client Management & Services

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