

Electrical Theory

Math Review

PJM State & Member Training Dept.

By the end of this presentation the Learner should be able to:

- Use the basics of trigonometry to calculate the different components of a right triangle
- Compute Per-Unit Quantities
- Identify the two components of Vectors



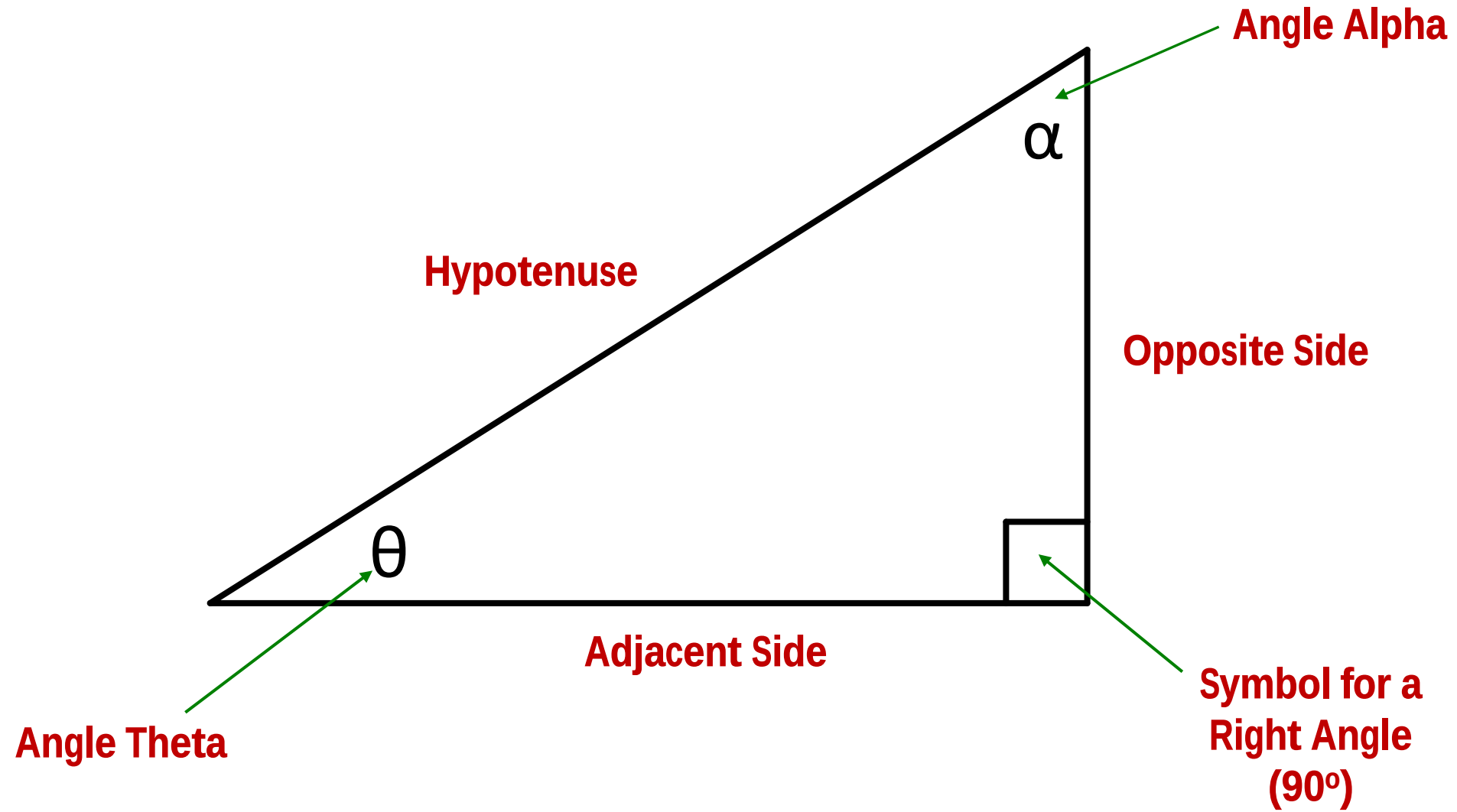
Right Triangles



Mathematics Review

- To be able to understand basic AC power concepts, a familiarization with the relationships between the angles and sides of a right triangle is essential
- A right triangle is defined as a triangle in which one of the three angles is equal to 90°

Mathematics Review



Mathematics Review

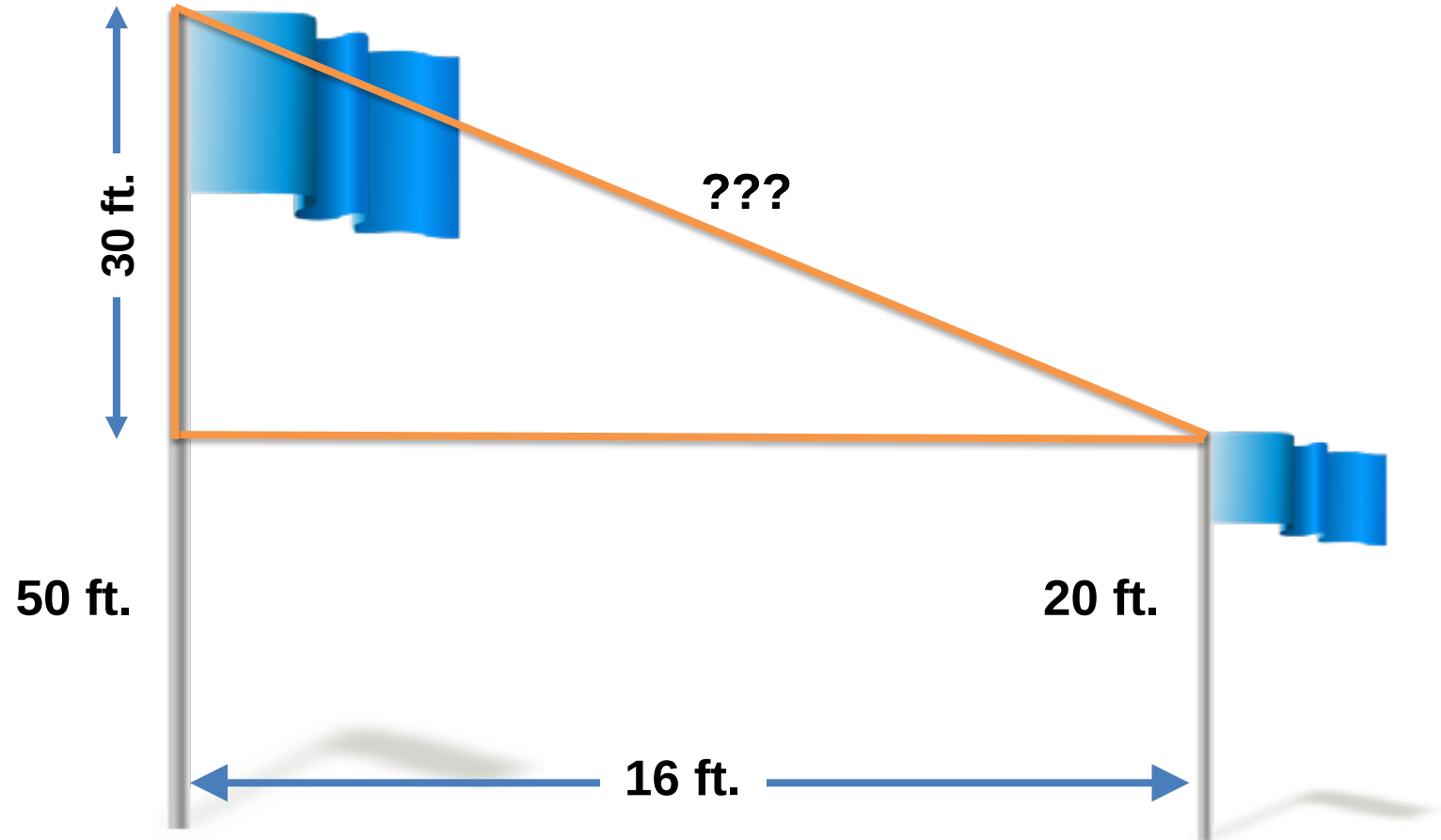
- Given the lengths of two sides of a right triangle, the third side can be determined using the [Pythagorean Theorem](#)

$$\text{Hypotenuse}^2 = \text{Opposite}^2 + \text{Adjacent}^2$$

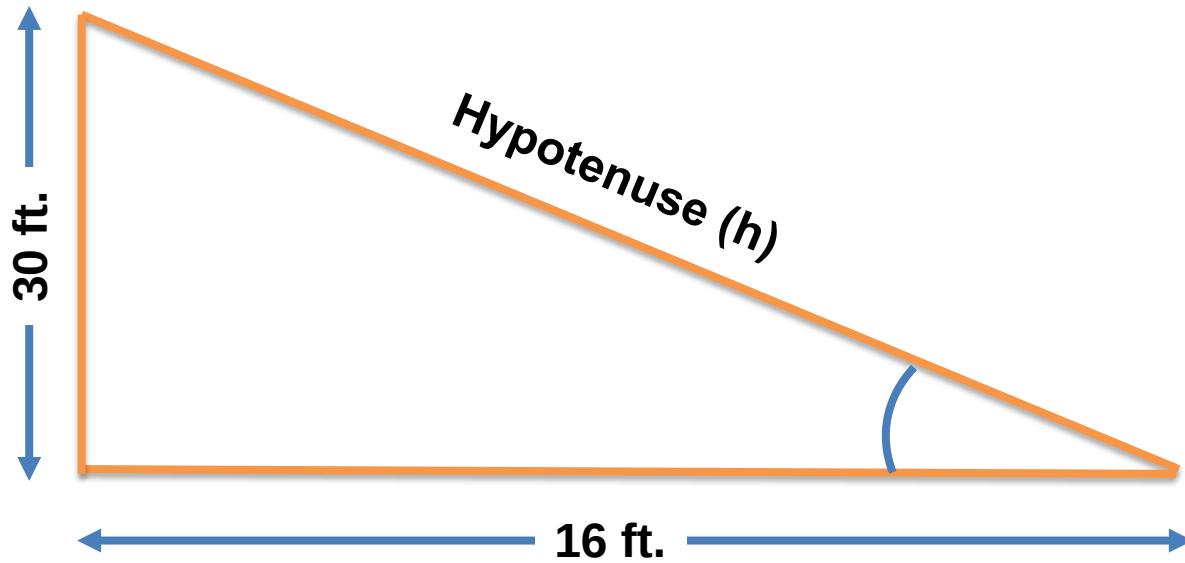
Example:

A rope stretches from the top of one pole 50 feet high to the top of another pole 20 feet high, standing 16 feet away.

How long is the rope?



Example:



$$\begin{aligned}h^2 &= a^2 + o^2 \\h^2 &= 16^2 + 30^2 \\h^2 &= 256 + 900 \\h^2 &= 1156 \\h &= \sqrt{1156} \\h &= 34\end{aligned}$$

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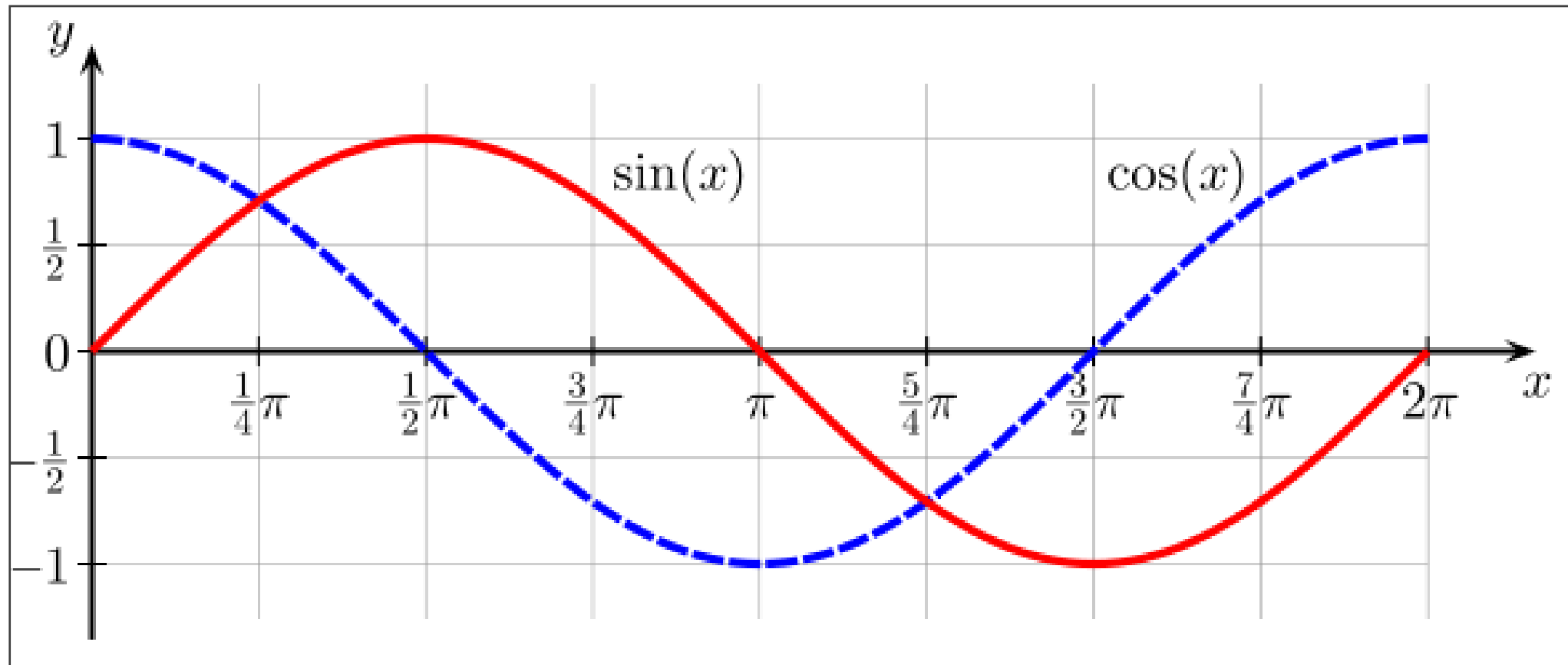
- Once the sides are known, the next step in solving the right triangle is to determine the two unknown angles of the right triangle
- All of the angles of any triangle always add up to 180°
- In solving a right triangle, the remaining two unknown angles must add up to 90°
- Basic trigonometric functions are needed to solve for the values of the unknown angles

Trigonometry



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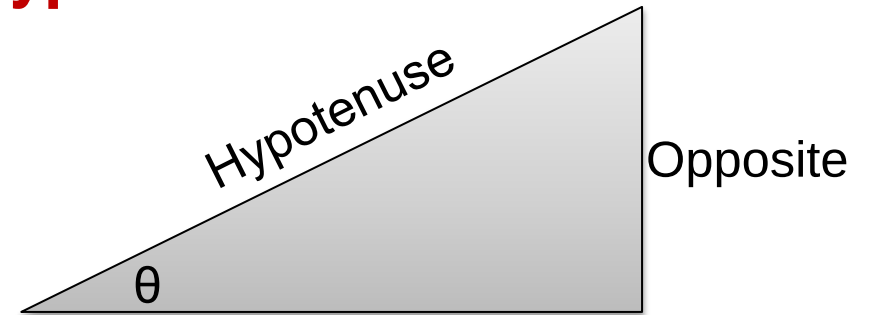
- The sine function is a periodic function in that it continually repeats itself



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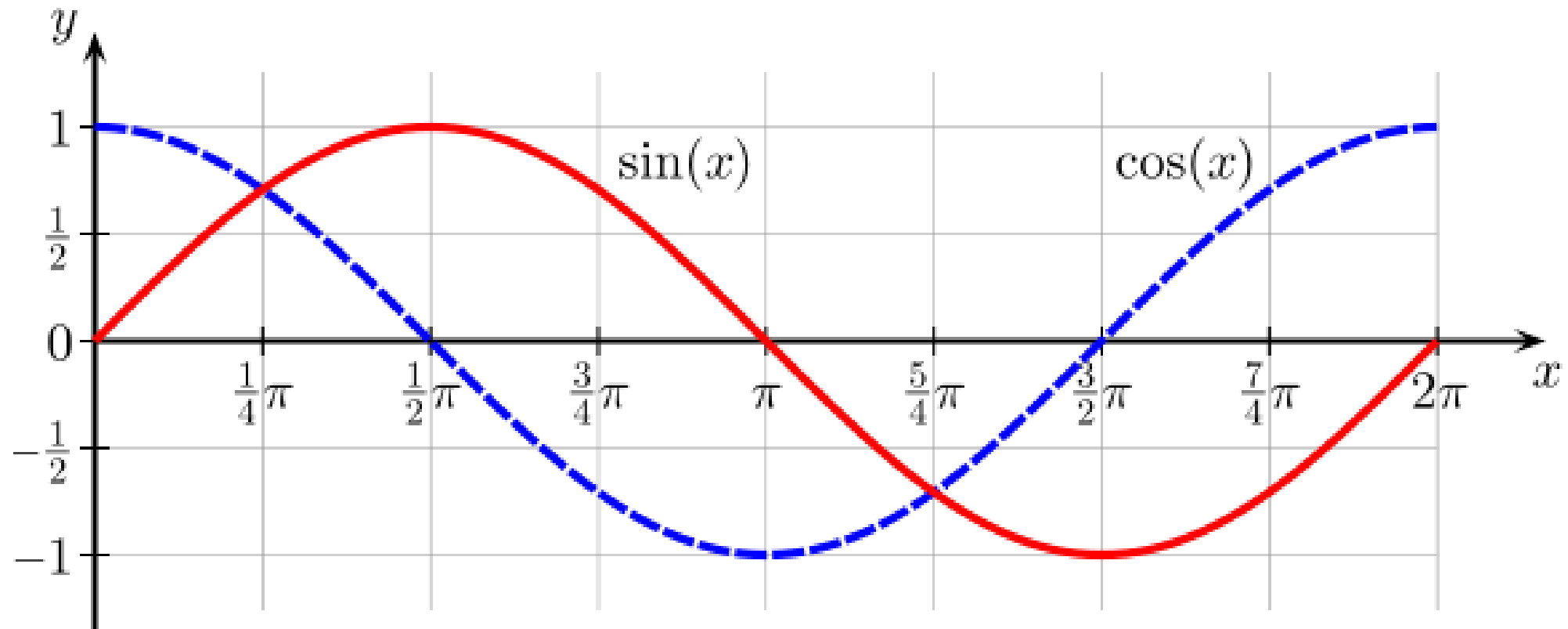
- In order to solve right triangles, it is necessary to know the value of the sine function between 0° and 90°
- Sine of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the hypotenuse

$$\text{SIN } \theta = \text{Opposite Side} / \text{Hypotenuse}$$



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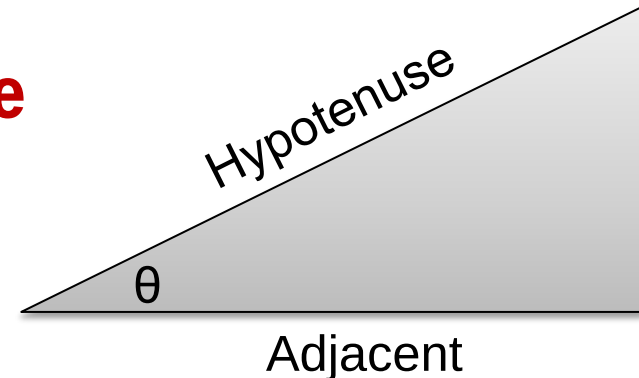
- Cosine function is a periodic function that is identical to the sine function except that it leads the sine function by 90°



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- As an example, the cosine function at 0° is 1 whereas the sine function does not reach the value of 1 until 90°
- Cosine function of either of the unknown angles of a right triangle is the ratio of the length of the adjacent side to the length of the hypotenuse

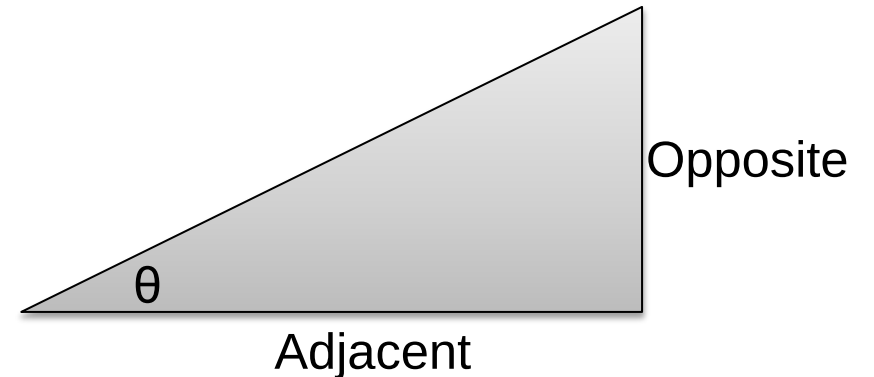
$$\text{COS } \theta = \text{Adjacent Side} / \text{Hypotenuse}$$



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- The tangent function of either of the unknown angles of a right triangle is the ratio of the length of the opposite side to the length of the adjacent side

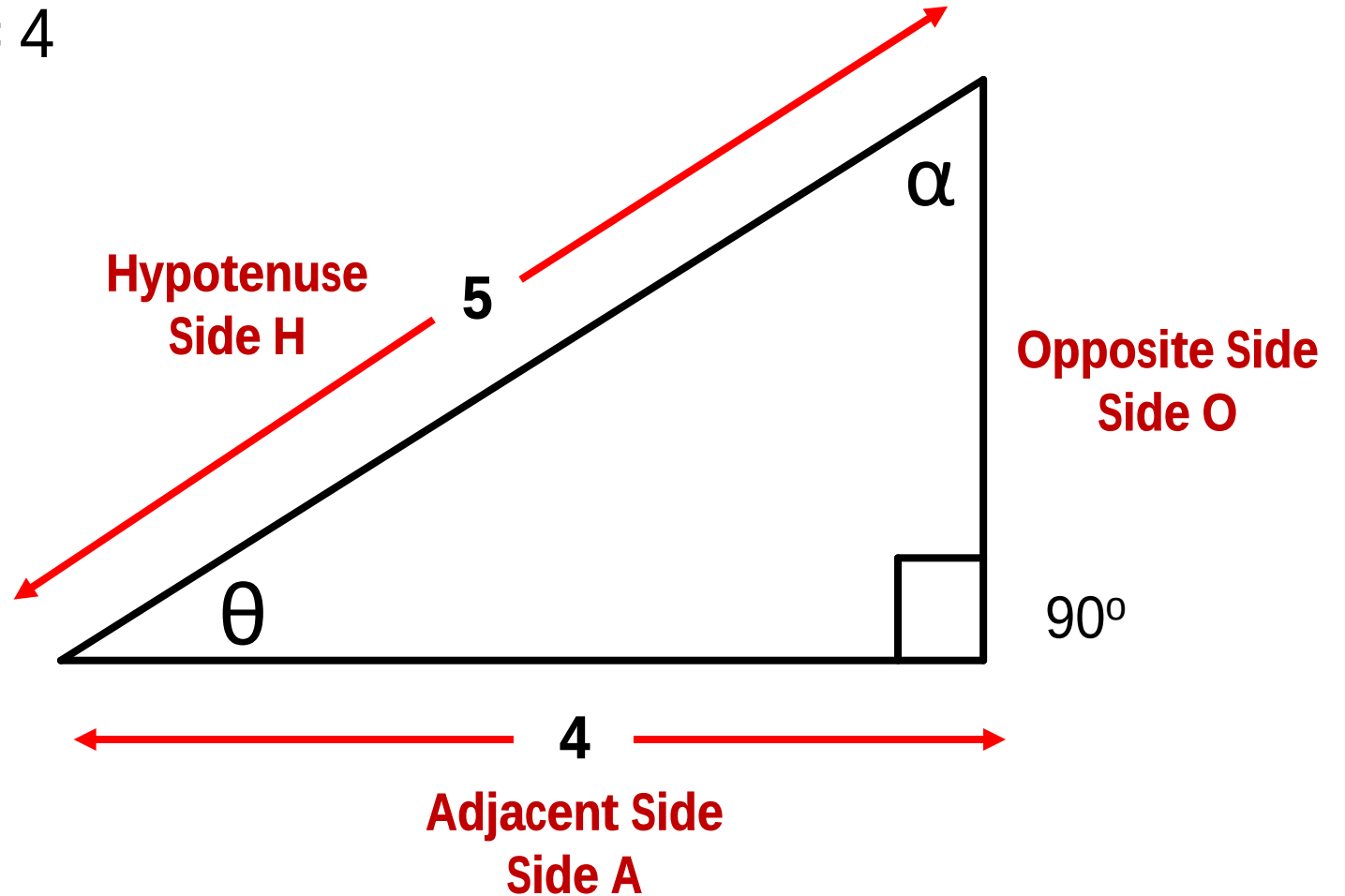
$$\text{TAN } \theta = \text{Opposite Side} / \text{Adjacent Side}$$



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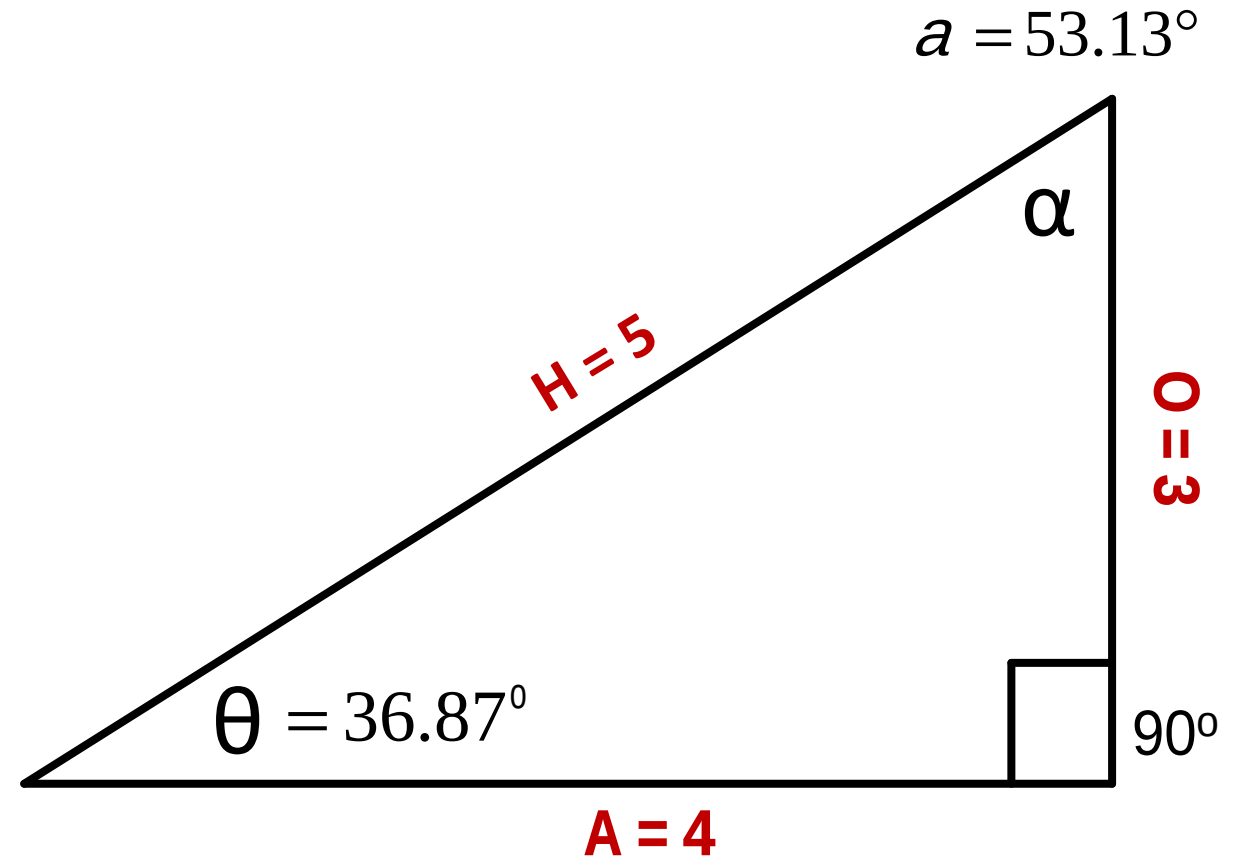
Example:

- Given: Side H = 5, Side A = 4
- Find: Side O, Angle q and Angle a



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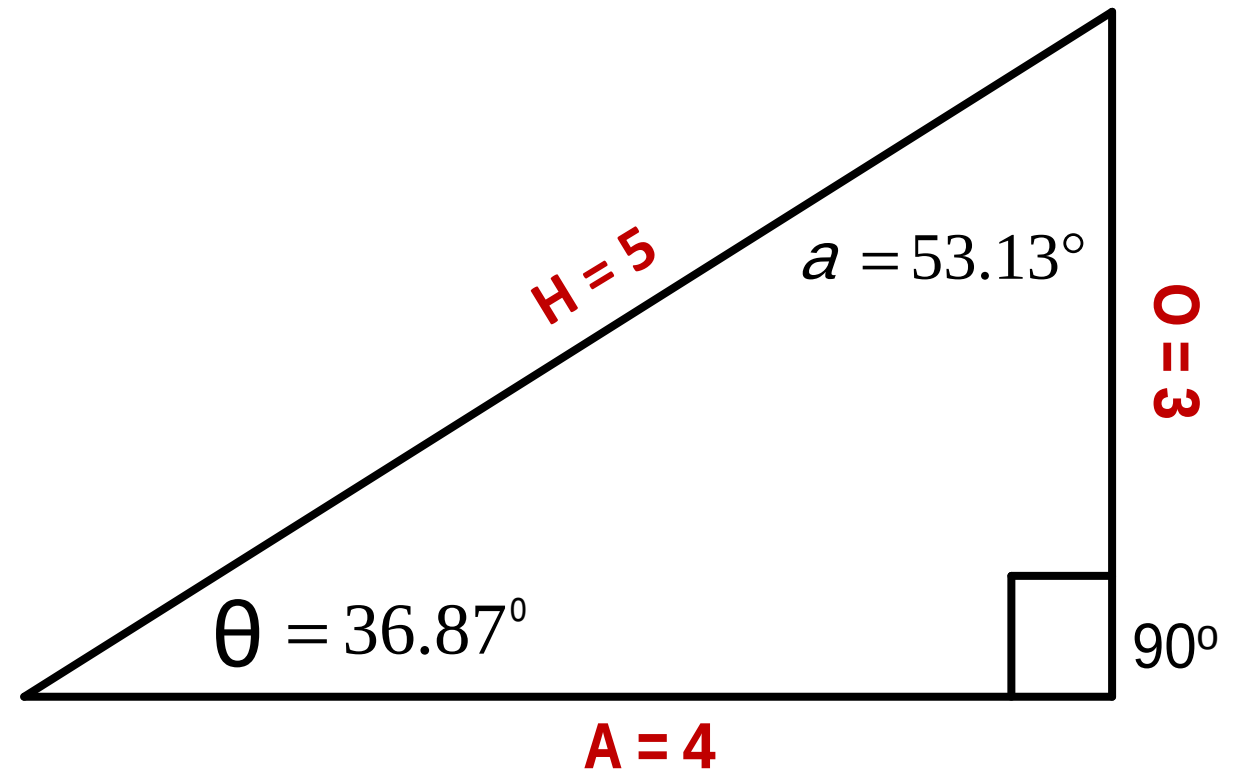
- Find Side O: $H^2 = A^2 + O^2$
 $25 = 16 + O^2$
 $25 - 16 = O^2$
 $\sqrt{25 - 16} = O$
 $O = \sqrt{9} = 3$
- Find α : $\cos \theta = \frac{A}{H} = \frac{4}{5} = .8$
 $\cos^{-1}(.8) = 36.87^\circ$
- Find a : $180^\circ - 90^\circ - 36.87^\circ = 53.13^\circ$



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$$\left. \begin{aligned} \theta &= \sin^{-1}\left(\frac{3}{5}\right) \\ \theta &= \cos^{-1}\left(\frac{4}{5}\right) \\ \theta &= \tan^{-1}\left(\frac{3}{4}\right) \end{aligned} \right\} 36.87$$

$$\left. \begin{aligned} \alpha &= \sin^{-1}\left(\frac{4}{5}\right) \\ \alpha &= \cos^{-1}\left(\frac{3}{5}\right) \\ \alpha &= \tan^{-1}\left(\frac{4}{3}\right) \end{aligned} \right\} 53.13$$



Per Unit Quantities



Mathematics Review

- Ratios play an important part in estimating power system performance
 - Relationship between two quantities as a fraction
 - Used when the relationship of two pairs of values is the same, and one of two similarly related values is known
 - Example: if you can drive 120 miles in 2 hours, how many miles could you drive in 8 hours?

$$\frac{120 \text{ miles}}{2 \text{ hours}} = \frac{X \text{ miles}}{8 \text{ hours}} = \frac{120 \text{ miles}(\cancel{8 \text{ hours}})}{2 \cancel{\text{hours}}} = \frac{X \text{ miles} (\cancel{8 \text{ hours}})}{\cancel{8 \text{ hours}}} = 480 \text{ miles}$$

Mathematics Review

- Quantities on the power system are often specified as a percentage or a **per-unit** of their base or nominal value
 - Makes it easier to see where a system value is in respect to its base value
 - How it compares between different parts of the system with different base values
 - Allow for a dispatcher to view the system and quickly obtain a feel for the voltage profile

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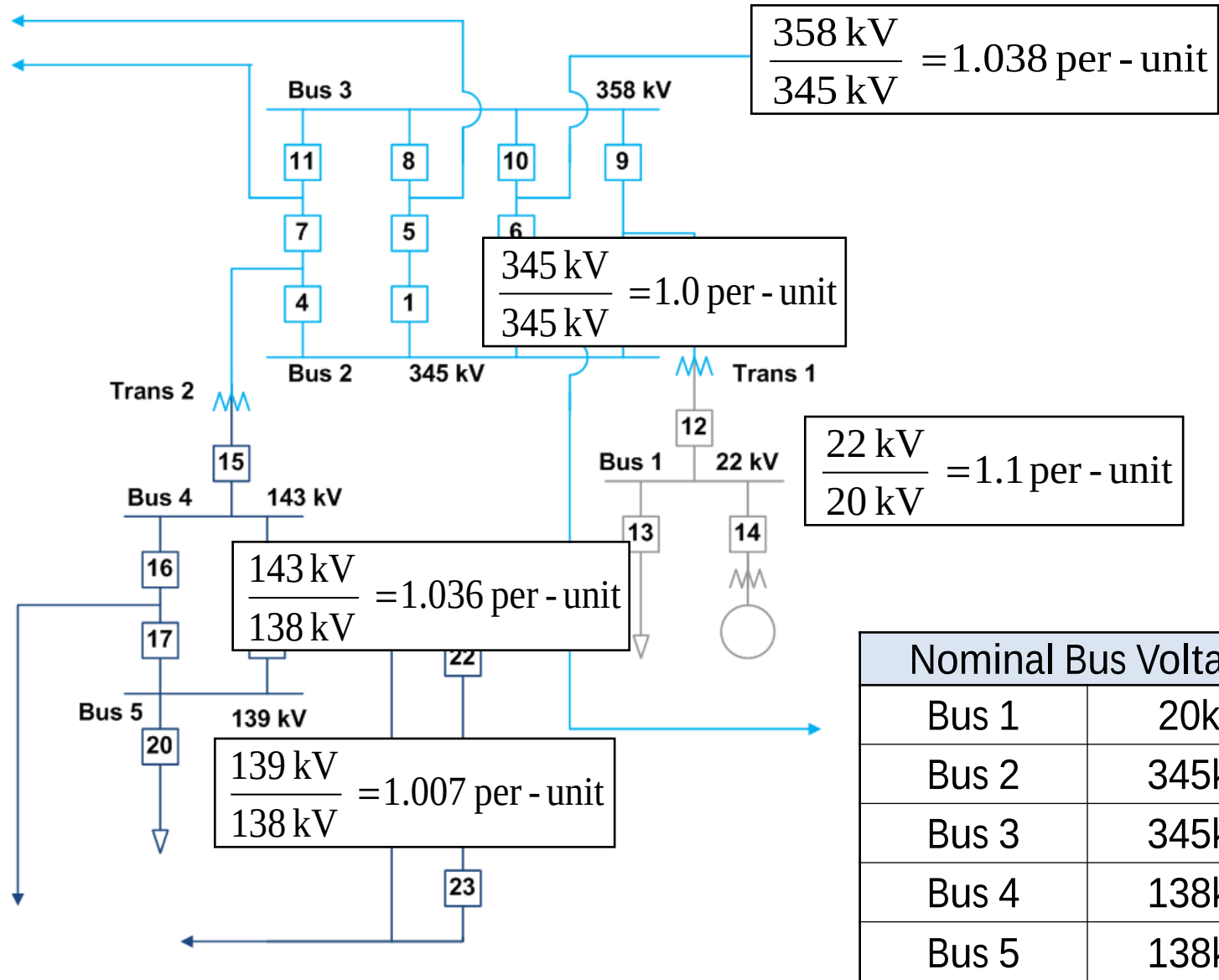
- Assume that, at a certain substation, the voltage being measured is 510 kV on the 500 kV system. What is its per-unit value with respect to the nominal voltage?

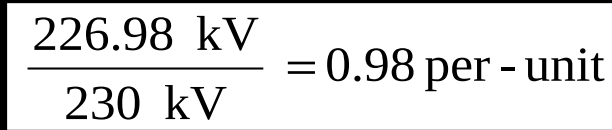
Base or nominal voltage = 500 kV

Measured voltage = 510 kV

$510 \text{ kV} / 500 \text{ kV} = \underline{1.02 \text{ per-unit}}$ or 102%

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$$\frac{513.9 \text{ kV}}{500 \text{ kV}} = 1.03 \text{ per - unit}$$


Nominal Bus Voltage	
East Bus	500 kV
West Bus	
North Bus	230 kV
South Bus	
Main Bus	115 kV

$$\frac{113.4 \text{ kV}}{115 \text{ kV}} = 0.98 \text{ per - unit}$$

Answer Question 4 & 5

Question 4

- Assume that the loss of a 1000 MW generating unit will typically result in a 0.2 Hz dip in system frequency
- Estimate the frequency dip for the loss of an 800 MW generating unit

Question

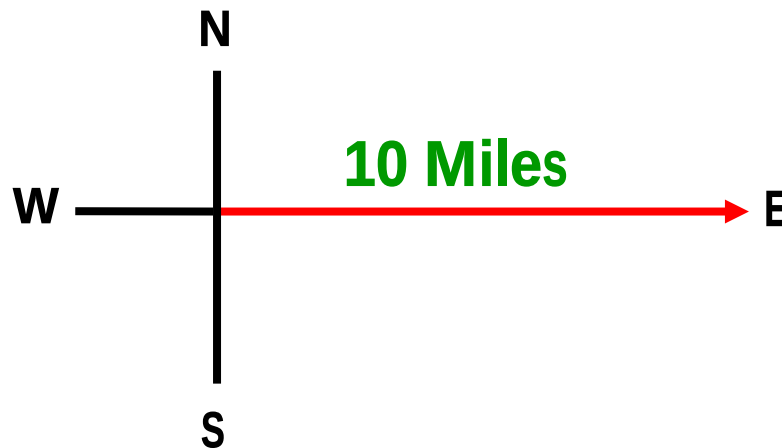
- Assume that the loss of a 500 MW generating unit will typically result in a 0.3 Hz dip in system frequency.
- Estimate the frequency dip for the loss of an 300 MW generating unit.

Vectors



Vectors

- A vector is alternative way to represent a sinusoidal function with amplitude, and phase information
- Vectors are usually written in bold with an arrow over the top of the letter, ($\vec{A}$)
- A vector's length represents **magnitude**
- A vector's **direction** represents the phase angle
 - Example: 10 miles east



Vectors

- Horizontal line to the right is positive; horizontal line to the left is negative
- Vertical line going up is positive; vertical line going down is negative
- Arrowhead on the end away from the point of origin indicates the direction of the vector and is called the displacement vector
- Vectors can go in any direction in space

Vectors

- The difference between a scalar quantity and a vector:
 - a) A scalar quantity is one that can be described with a single number, including any units, giving its size or magnitude
 - b) A vector quantity is one that deals inherently with both magnitude and direction

Conceptual Question 6

- There are places where the temperature is $+20^{\circ}\text{C}$ at one time of the year and -20°C at another time.
 - Do the plus and minus signs that signify positive and negative temperatures imply that temperature is a vector quantity?

Question 7

Which of the following statements, if any, involves a vector?

- a) I walked two miles along the beach
- b) I walked two miles due north along the beach
- c) A ball fell off a cliff and hit the water traveling at 17 miles per hour
- d) A ball fell off a cliff and traveled straight down 200 feet
- e) My bank account shows a negative balance of -25 dollars

Vectors

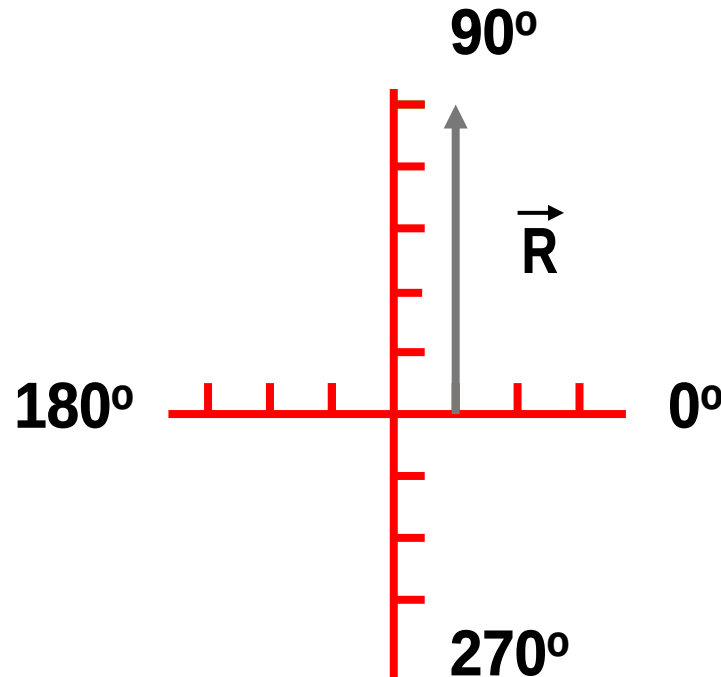
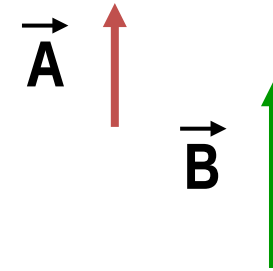
- When adding vectors, the process must take into account both the magnitude and direction of the vectors
- When adding two vectors, there is always a resultant vector, R , and the addition is written as follows:

$$\vec{R} = \vec{A} + \vec{B}$$

Vectors

Example: Adding vectors in the same direction

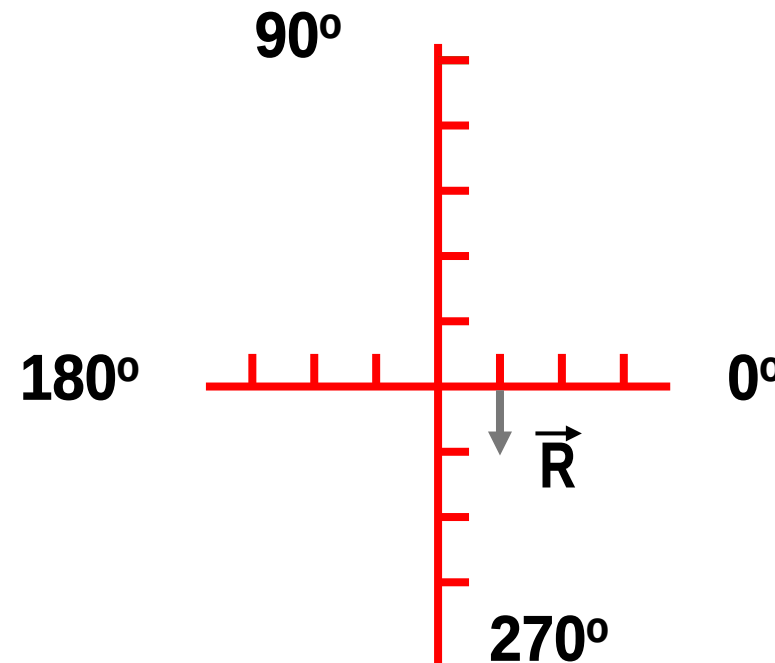
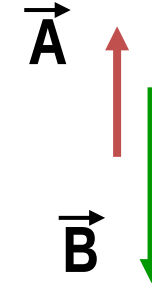
- Vector $\vec{A}$ has a length of 2 and a direction of 90°
- Vector $\vec{B}$ has a length of 3 and a direction of 90°



Vectors

Example: Adding vectors in the opposite direction

- Vector $\vec{A}$ has a length of 2 and a direction of 90°
- Vector $\vec{B}$ has a length of 3 and a direction of 270°



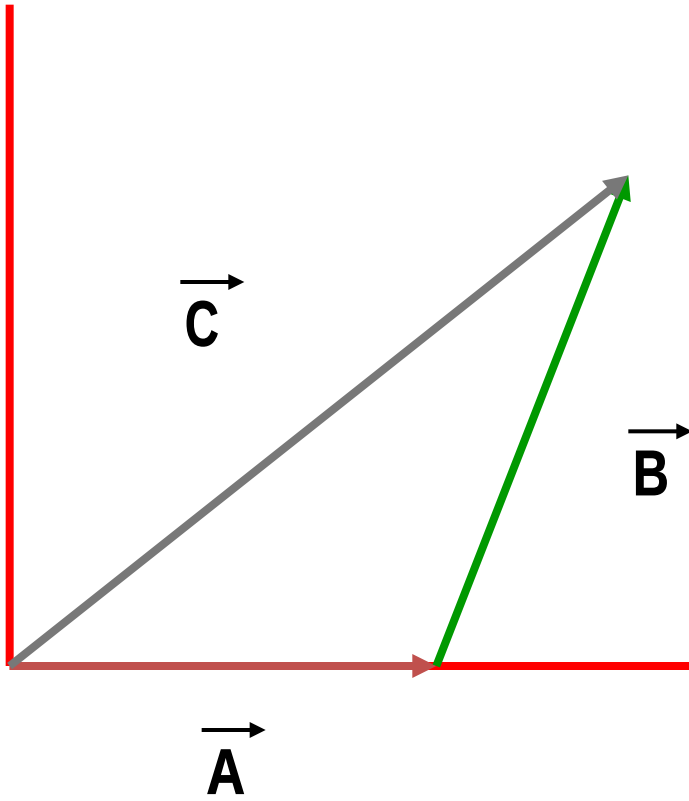
Vectors

- Subtraction of one vector from another is carried out in a way that depends on the following:

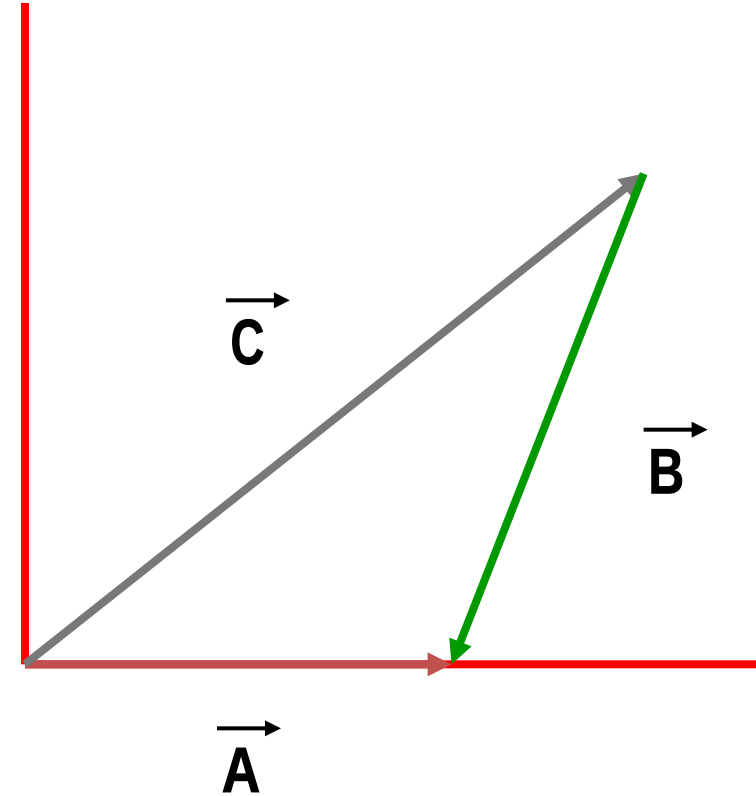
When a vector is multiplied by -1 , the magnitude of the vector remains the same, but the direction of the vector is reversed

- Vector subtraction is carried out exactly like vector addition except that one of the vectors added is multiplied by the scalar factor of -1

Vectors



$$\vec{C} = \vec{A} + \vec{B}$$



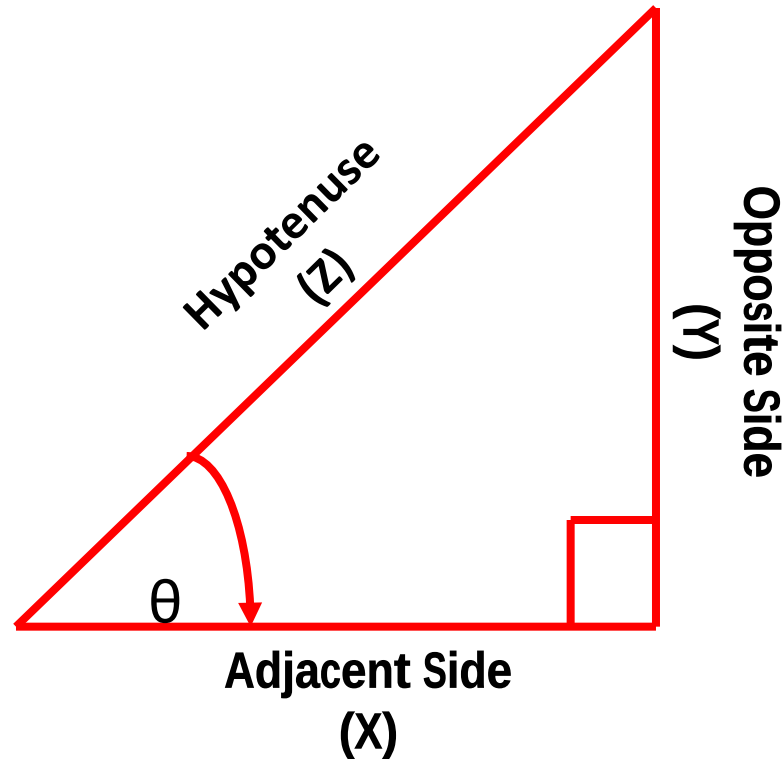
$$\vec{A} = \vec{C} + (-\vec{B})$$

Vectors

- If the magnitude and direction of a vector is known, it is possible to find the components of the vector
- The process is called “resolving the vector into its components”
- If the vector components are perpendicular and form a right triangle, the process can be carried out with the aid of trigonometry

Vectors

- To calculate the sum of two or more vectors using their components (x and y) in the vertical and horizontal directions, trigonometry is used



Vectors

- The Pythagorean Theorem is a special relationship that exists in any triangle and describes the relationship between the lengths of the sides of a right triangle

$$z^2 = x^2 + y^2$$

- Three basic trigonometric functions defined by a right triangle are:

$$\sin \theta = y/z = \text{opposite side/hypotenuse}$$

$$\cos \theta = x/z = \text{adjacent side/hypotenuse}$$

$$\tan \theta = y/x = \text{opposite side/adjacent side}$$

$$\tan \theta = \sin \theta / \cos \theta$$

Vectors

- To find theta, the inverse of the trigonometric function must be used

$$\theta = \sin^{-1} y/z$$

$$\theta = \cos^{-1} x/z$$

$$\theta = \tan^{-1} y/x$$

- When adding vectors, magnitude is found by:

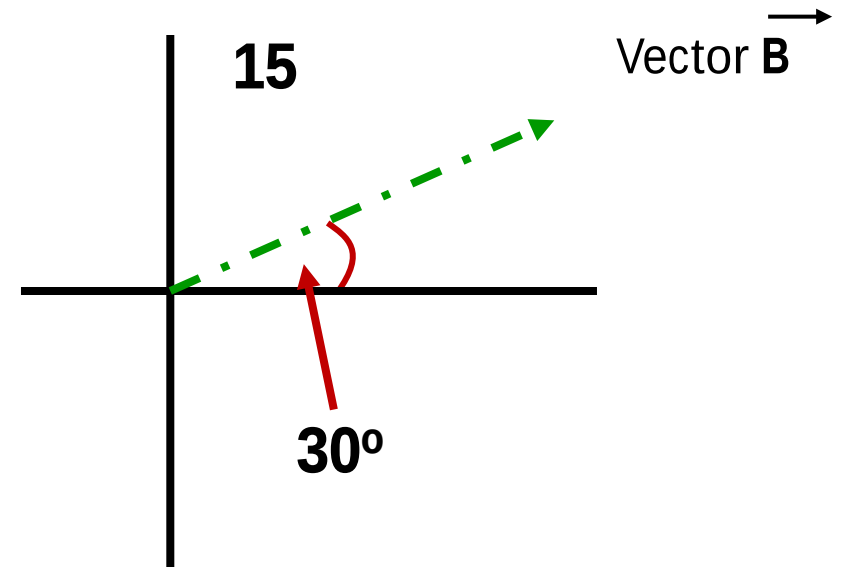
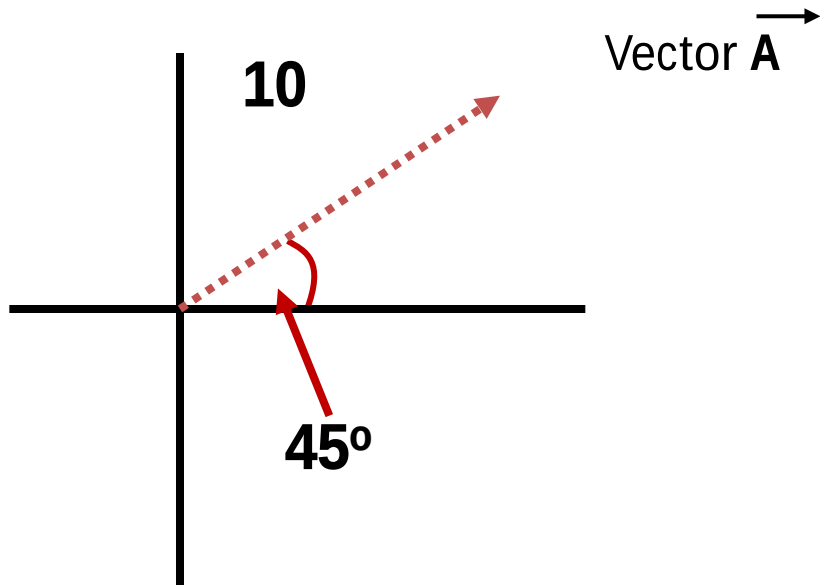
$$\vec{R} = \sqrt{(\vec{R}_x)^2 + (\vec{R}_y)^2}$$

- The direction of the resultant, R, is found by:

$$\theta = \sin^{-1} (R_y/R)$$

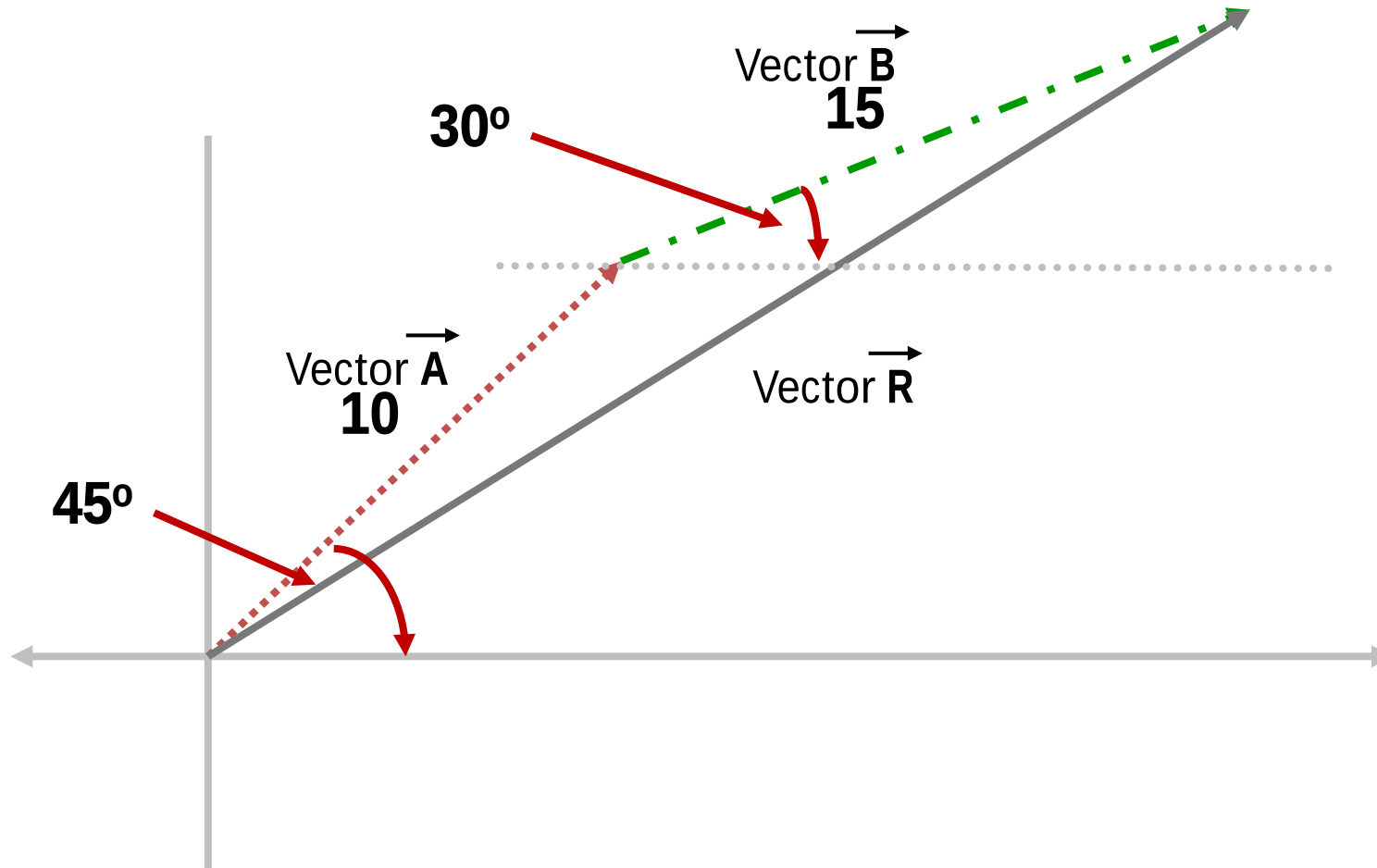
Vectors

- Vectors can be added either in the same direction or in opposite directions

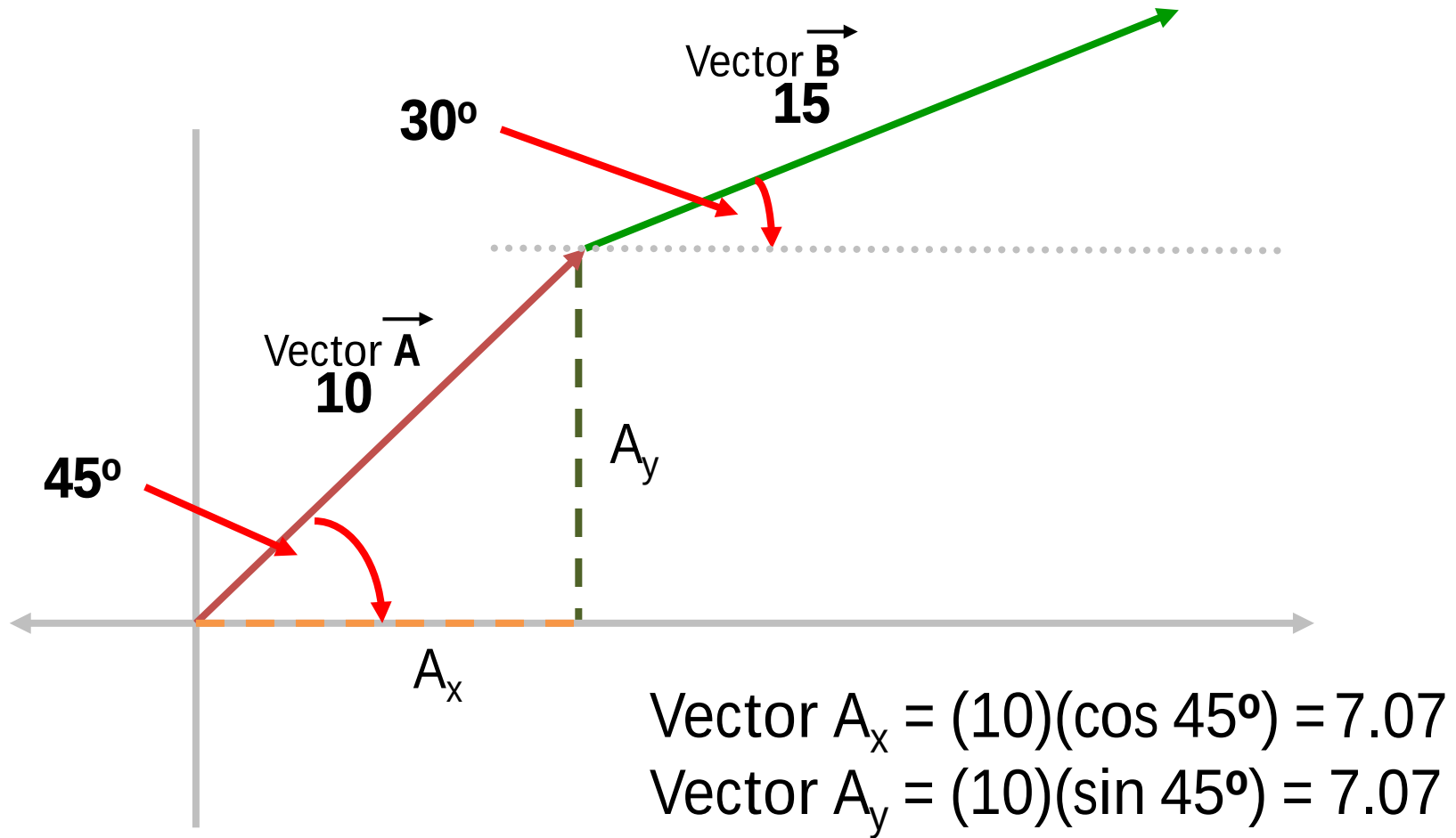


Vectors

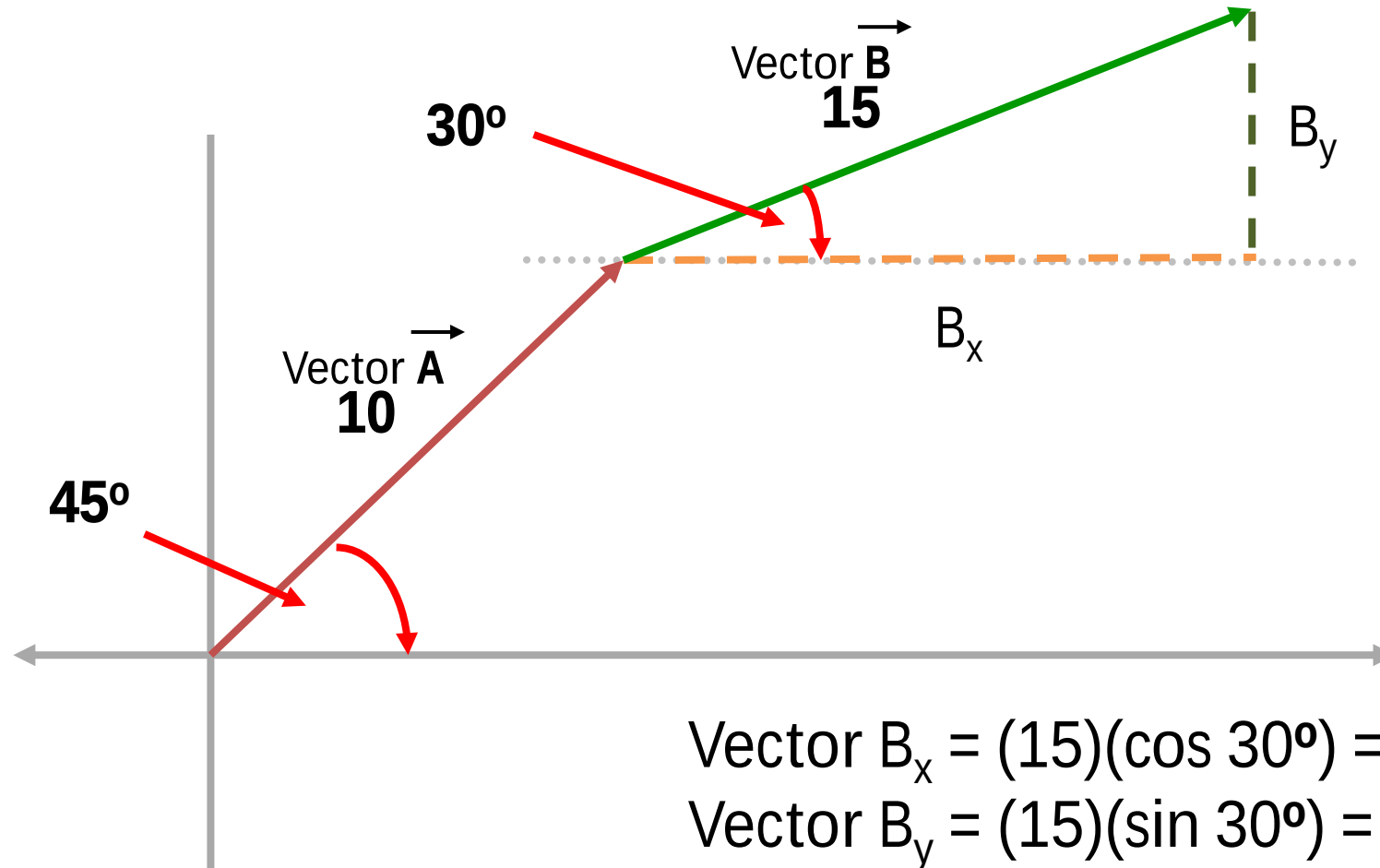
Adding vectors:



Vectors



Vectors



Vectors

Determine the resulting vectors, R_x and R_y :

$$\overrightarrow{R_x} = \overrightarrow{A_x} + \overrightarrow{B_x}$$

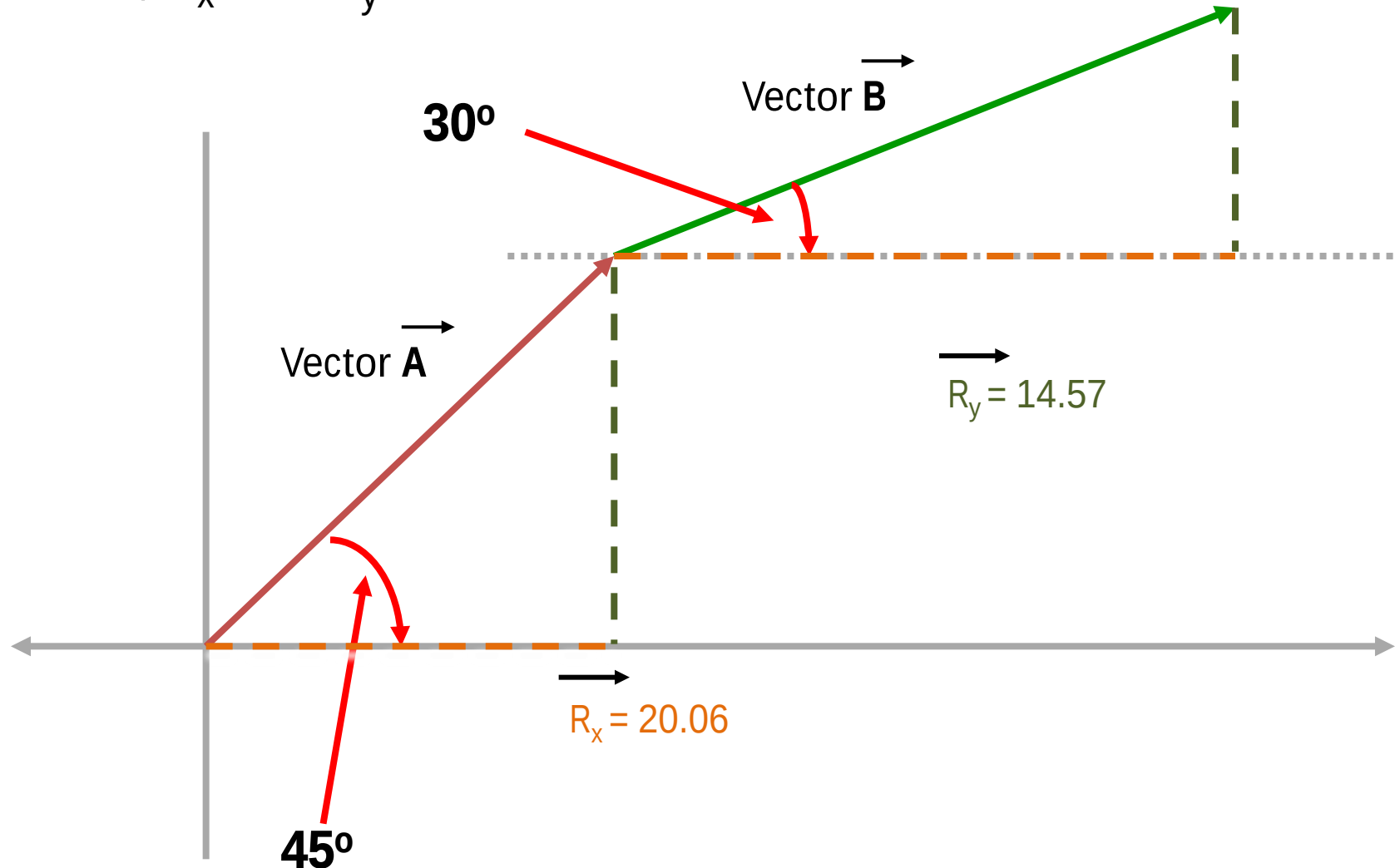
$$\overrightarrow{R_x} = 7.07 + 12.99$$

$$\overrightarrow{R_x} = 20.06$$

$$\overrightarrow{R_y} = \overrightarrow{A_y} + \overrightarrow{B_y}$$

$$\overrightarrow{R_y} = 7.07 + 7.5$$

$$\overrightarrow{R_y} = 14.57$$



Vectors

$$R = \sqrt{R_x^2 + R_y^2}$$

$$R = \sqrt{20.06^2 + 14.57^2}$$

$$R = \sqrt{614.7}$$

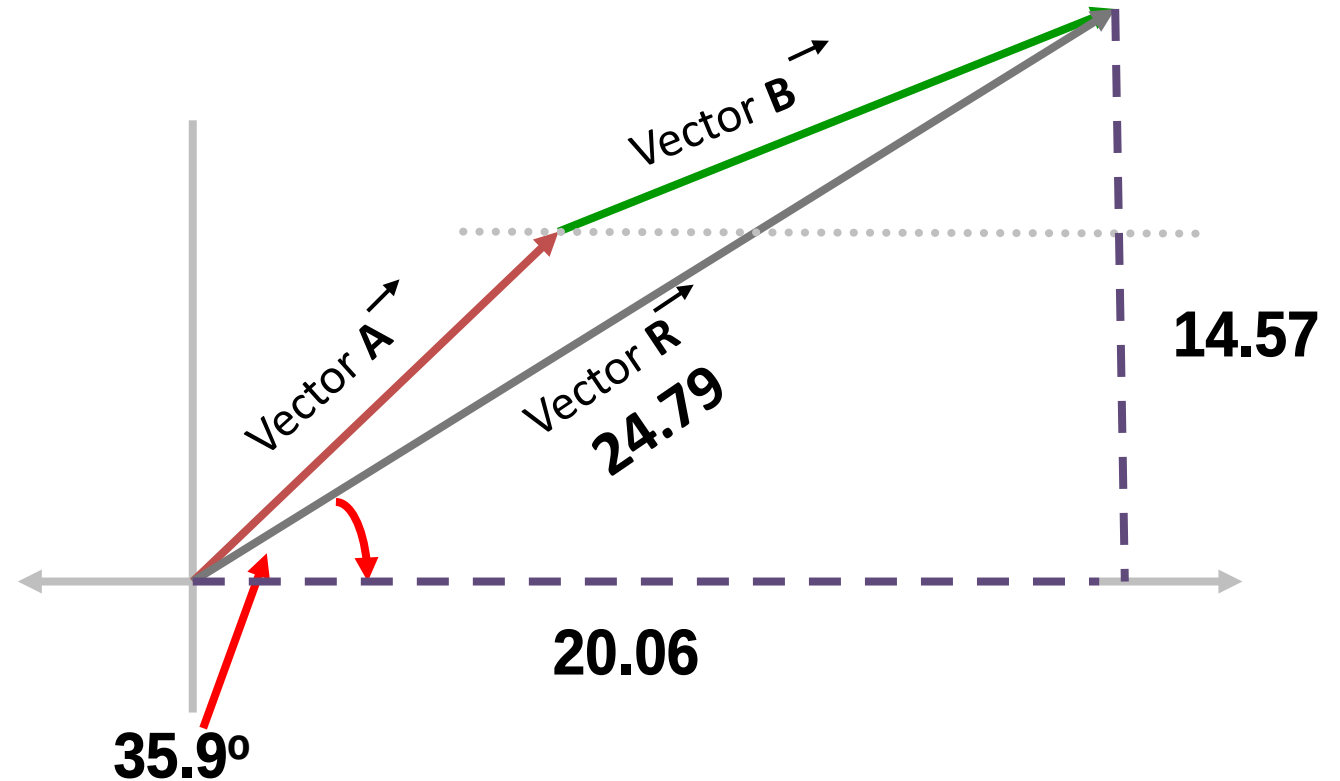
$$R = 24.79$$

$$q = \sin^{-1} \frac{R_y}{R}$$

$$q = \sin^{-1} \frac{14.57}{24.79}$$

$$q = \sin^{-1}(.587)$$

$$q = 35.9^\circ$$



Vectors

- Polar notation expresses a vector in terms of both a magnitude and a direction, such as:

$$\mathbf{M} \angle ^{\circ}$$

where:

M is the magnitude of the vector

[°] is the direction in degrees

Example:

Vector with a magnitude of 10 and a direction of -40 degrees $10 \angle -40^{\circ}$

Vectors

- Multiplication in polar notation:

Multiply the magnitudes/add the angles

$$(50 \angle 25^\circ) + (25 \angle 30^\circ) = 1250 \angle 55^\circ$$

- Division in polar notation:

Divide the magnitudes/subtract the angles

$$\frac{(50 \angle 25^\circ)}{(25 \angle 30^\circ)} = 2 \angle -5^\circ$$

Questions?

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