Energy and Ancillary Service Co-Optimization Formulation

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I. Introduction

This document provides a high-level mathematical formulation and explanation of the objective function used in the clearing of the Energy and Ancillary Service Markets. The intent of this document is to provide a better understanding of the mechanics.

Due to the approaching implementation of Reserve Price Formation, the paper provides details on the Secondary Reserve market. Such costs and constraints will not be considered until the implementation, scheduled for 10/1/2022.

II. Security Constraint Economic Dispatch (SCED) Objective Function

SCED is a mathematical model that generates the most economic resource dispatch during Real-time operations while considering key system operating constraints, such as power balance, reserve requirements, transmission congestion, as well as resource parameters, such as ramp rates, minimum and maximum output capability. The overall objective function of the SCED algorithm is to minimize the total system product cost over the study interval(s).

MINIMIZE {Resource Energy Costs
 - Price Responsive Demand Value

+ Import Transaction Cost

- Export Transaction Value

+ Regulation Reserve Costs

+ Synchronized Reserve Costs

+ Non-Synchronized Reserve Costs

+ Secondary Reserve Costs

+ Various Applicable Violation Penalties***}**

*Note: Includes transmission constraint violation penalties, energy surplus and deficit penalties, regulation and reserves violation penalties, etc.

The objective function cost terms are described below. For purposes of the following equations, the parameters are defined as:

- *n = Number of Resources*
- *PRD = Price Responsive Demand*
- *ImportTran. = Number of import transactions*
- *ExportTran. = Number of export transactions*
- *RegResource = Number of Regulation eligible resources*
- *SRResource = Number of eligible Synchronized Reserve resources*
- *NSRResource = number of resources eligible for Non-Synchronized Reserve and not committed for energy*

- *SecRResource = Number or resources eligible for Secondary Reserve*
- *LS = Loss Sensitivities*

A. Resource Energy Costs

Resource energy costs apply only to generation resources and Economic Load Response resources. Energy costs are expressed mathematically in the SCED objective function as follows:

Resource Energy Costs

$$\sum_{i=1}^{n} Energy\_MW(i) * EnergyOfferCurve(i) \quad for\ \forall\ i \in n\ number\ of\ resources$$

B. Price Responsive Demand Value

Price Responsive Demand value is expressed mathematically in the SCED objective function as follows:

Price Responsive Demand Value

$$\sum_{i=1}^{PRD} PRD\_MW(i) * EnergyOfferCurve(i) \quad for\ \forall\ i \in PRD\ resources$$

C. Import Transaction Costs

Import transaction cost is expressed mathematically in the SCED objective function as follows:

Import Transaction Cost

$$\sum_{i=1}^{ImportTran.} Import\_Transaction\_\ MW(i) * Import\ Transaction\ offer\ Curve(i) \quad for\ \forall\ i \in ImportTran.$$

D. Export Transaction Value

Export value is expressed mathematically in the SCED objective function as follows:

Export Transaction Value

$$\sum_{i=1}^{ExportTran.} Export\_Transaction\_\ MW(i) * Export\ Transaction\ Bid\ Curve(i) \quad for\ \forall\ i \in ExportTran.$$

E. Regulation Reserve Costs

Regulating reserve costs apply only to regulation-eligible resources. Regulating reserve costs are expressed mathematically in the SCED objective function as follows:

$$\sum_{i=1}^{RegResource} Reg\_MW(i) * RegOffer(i) \quad for\ \forall\ i \in RegResource$$

F. Synchronized Reserve Costs

Synchronized Reserve costs apply to all synchronized reserve eligible resources. Synchronized Reserve costs are expressed mathematically in the SCED objective function as follows:

$$\sum_{i=1}^{SRResource} SR\_MW(i) * SRofferCurve(i) \quad for\ \forall\ i \in SRResource$$

G. Non-Synchronized Reserve Costs

Non-Synchronized Reserve costs apply to all non-synchronized reserve eligible resources. Non-Synchronized Reserve costs are expressed mathematically in the SCED objective function as follows:

$$\sum_{i=1}^{NSRResource} NSR\_MW(i) * NSRofferCurve(i) \quad for\ \forall\ i \in NSRResource$$

H. Secondary Reserve Costs

Secondary Reserve costs apply to all synchronized reserve eligible resource. Secondary Reserve costs are expressed mathematically in the SCED objective function as follows:

$$\sum_{i=1}^{SecRResource} SecR\_MW(i) * SecRofferCurve(i) \quad for\ \forall\ i \in SecRResource$$

III. SCED Constraints

The objective function of SCED is minimized subject to the following constraints in optimizations.

A. Power Balance Constraint

The power balance constraint in SCED ensures that all energy injected into the PJM system during the dispatch interval balances with all energy withdrawal including losses in the system.

$$\sum_{i=1}^{n}(1-LS(i))*Energy_{MW(i)}+\sum_{i=1}^{importTran.}(1-LS(i))*\ Import_{Transaction_{MW(i)}}-\ \sum_{i=1}^{ExportTran.}(1-LS(i))*Export_{Transaction_{MW(i)}}-\sum_{i=1}^{PRD}(1$$
$$=Load\ Forecast$$

B. Transmission Constraints

The transmission constraints in SCED ensure that the flows over the monitored transmission constraints are within the applicable limits of transmission lines.

$$\sum_{i=1}^{n}Energy\_MW(i)*DFax(i,k)+\sum_{i=1}^{importTran.}Import\_Transaction\_MW(i)*DFax(i,k)-\sum_{i=1}^{ExportTran.}Export\_Transaction\_MW(i)*DFax(i,k)-\sum_{i=1}^{PRD}PRD\_MW(i)*DFax(i,k)\leq Line\ Limit\ ;for\ \forall\ k\ \in active\ Transmission\ constraints$$

C. Resource Capacity Constraints

1. Resource's Economic Maximum Constraint Limit

$$\sum_{i=1}^{n}Energy\_MW(i)+SR\_MW(i)+SecR\_MW(i)+Reg\_MW(i)\leq EcoMax(i)\quad ;\quad for\ \forall\ i\in n, where\ n\in committed\ r$$

2. Resource's Economic Minimum Constraint Limit

$$\sum_{i=1}^{n}Energy\_MW(i)+Reg\_MW(i)\geq EcoMin(i)\ ;\quad for\ \forall\ i\in n\ , where\ n\in committed\ resources$$

3. Resource's Reserve Capability Constraints

Resource's Synchronized Reserve Capability Constraints

$$\sum_{i=1}^{n} SR\_MW(i) \leq RampRate(i) * 10 \quad ; \quad for\ \forall\ i \in n\ , where\ n \in committed\ resources$$

Resource's Non-Synchronized Reserve Capability Constraints

$$\sum_{i=1}^{n} NSR\_MW(i) \leq EcoMin(i) + (10 - startup\ plus\ notificationtime) * RampRate(i) \quad ; \quad for\ \forall\ i \in n\ , where\ n \in eli$$

Resource's Online Secondary Reserve Capability Constraints

$$\sum_{i=1}^{n} SecR_{MW(i)} \leq RampRate(i) * 20 \quad ; \quad for\ \forall\ i \in n\ , where\ n \in committed\ resources$$

Resource's Offline Secondary Reserve Capability Constraints

$$\sum_{i=1}^{n} SecR\_MW(i) \leq RampRate(i) * 20 \quad ; \quad for\ \forall\ i \in n\ , where\ n \in offline\ resources\ with\ startup\ plus\ notification$$

$$\sum_{i=1}^{n} SecR\_MW(i) \leq EcoMin(i) + (30 - startup\ plus\ notificatin\ time) * RampRate(i) \quad ; \quad for\ \forall\ i \in n\ , where\ n \in of$$

Resource's Regulation Capability Constraints

$$\sum_{i=1}^{n} Reg\_MW(i) \le Min[0.5 * (RegMax(i) - RegMin(i)), RegOfferMW(i)\,] \;\; ; \;\; for\ \forall\ i \in n\ , where\ n \in Regulation\ E$$

D. Resource's Ramp Rate Constraints

1. Resource's Ramp-Up Constraints

$$\sum_{i=1}^{n} Energy\_MW(i) \le Initial\_MW(i) + RampRate(i) * 5 \quad ;\ for\ \forall\ i \in n, \\ where\ n \in committed\ resources$$

2. Resource's Ramp-Down Constraints

$$\sum_{i=1}^{n} Initial\_MW(i) - Energy\_MW(i) \ge RampRate(i) * 5\ ; \quad for\ \forall\ i \in n, \\ where\ n \in committed\ resources$$

E. Reserve Requirement Constraints

1. Synchronized Reserve Requirement Constraints

For RTO reserve requirement:

$$\sum_{i=1}^{n} SR\_MW(i) \ge RTO\_SR\_Reserve\_Requirement \quad ;for\ \forall\ i \in n\ no.of\ resources;$$

For each sub-zone z:

$$\sum_{i=1}^{n} SR\_MW(i) \\ \ge Subzone\_SR\_Reserve\_Requirement \quad ;for\ \forall\ i \in n\ no.of\ resources, n \\ \in subzone\ z;$$

2. Primary Reserve Requirement Constraints

For RTO Primary Reserve Requirement:

$$\sum_{i=1}^{n} SR\_MW(i) + NSR\_MW(i) \geq RTO\_NSR\_Reserve\_Requirement \quad ;for\ \forall\ i \in n\ no.of\ resources;$$

For each sub-zone z:

$$\sum_{i=1}^{n} SR\_MW(i) + NSR\_MW(i) \geq Subzone\_NSR\_Reserve\_Requirement \quad ;for\ \forall\ i \in n\ no.of\ resources, n \in subzone\ z;$$

3. 30-minute Reserve Requirement Constraints

The RTO 30-minute Reserve Requirement is calculated as:

$$\sum_{i=1}^{n} SR\_MW(i) + NSR\_MW(i) + SecR\_MW(i) \geq RTO\_SecR\_Reserve\_Requirement \quad ;for\ \forall\ i \in n\ no.of\ resources;$$

For each sub-zone z:

$$\sum_{i=1}^{n} SR\_MW(i) + NSR\_MW(i) + SecR\_MW(i) \geq Subzone\_SecR\_Reserve\_Requirement \quad ;for\ \forall\ i \in n\ no.of\ resources, n \in subzone\ z;$$

IV. LMP and Reserve Clearing Price Calculations

Each Pnode LMP is calculated as:

$$LMP(i) = Energy\ Price - Loss\ Price(i) + Congestion\ Price(i) \qquad For\ for\ \forall\ i \in no.\ of\ pnodes$$

$$where,$$

$$Energy\ Price = Shadow\ Price\ of\ Power\ Balance\ Constraint$$

$$Loss\ Price\ (i) = LS(i) * Energy\ Price$$

$$Congestion\ Price(i) = \sum_{k=1}^{n} Shadow\ Price(k) * DFax(i,k) \qquad For\ \forall\ k \in active\ transmission\ constraints$$

A. Reserve Clearing Price

- Secondary Reserve Clearing Price = SP30
- Non-Synchronized Reserve Clearing Price = SPPR + SP30
- Synchronized Reserve Clearing Price = SPSR + SPPR + SP30

Where,

SP30 = Shadow Price of 30-minute Reserve Requirement

SPPR = Shadow Price of Primary Reserve Requirement

SPSR = Shadow Price of Synchronized Reserve Requirement